SUPPLEMENTAL MATERIALS

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Small-Scale Irrigation: Improving Food Security under Changing Climate and Water Resource Conditions in Ethiopia

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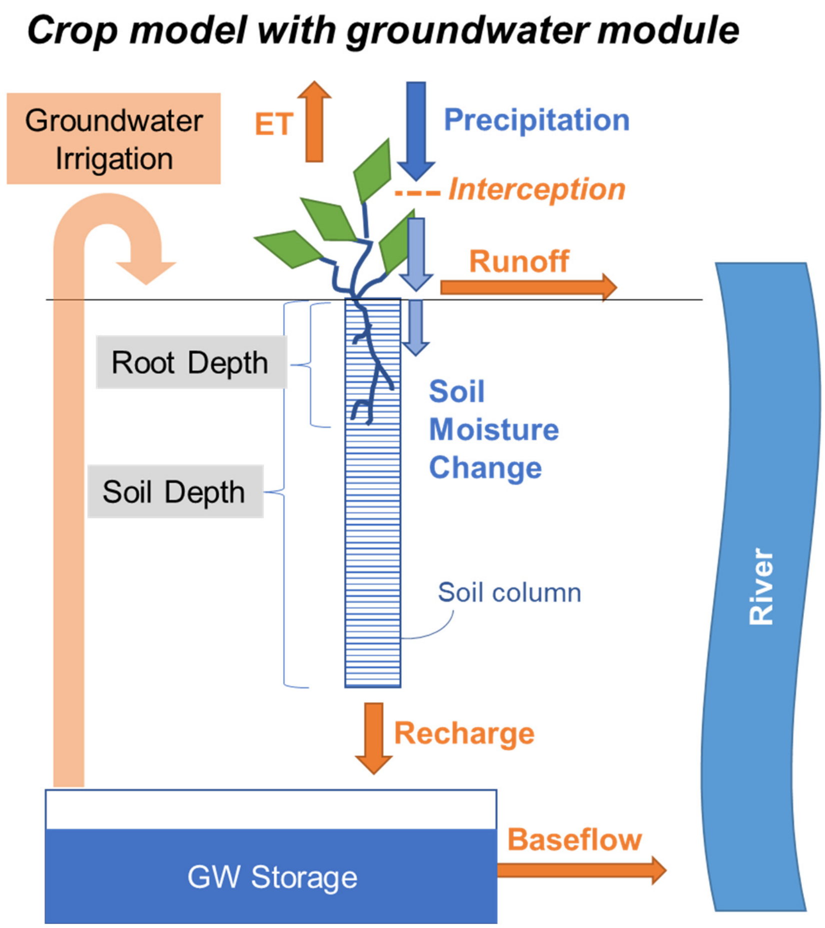

Fig. S1. Simplified illustration of the crop model with groundwater module.

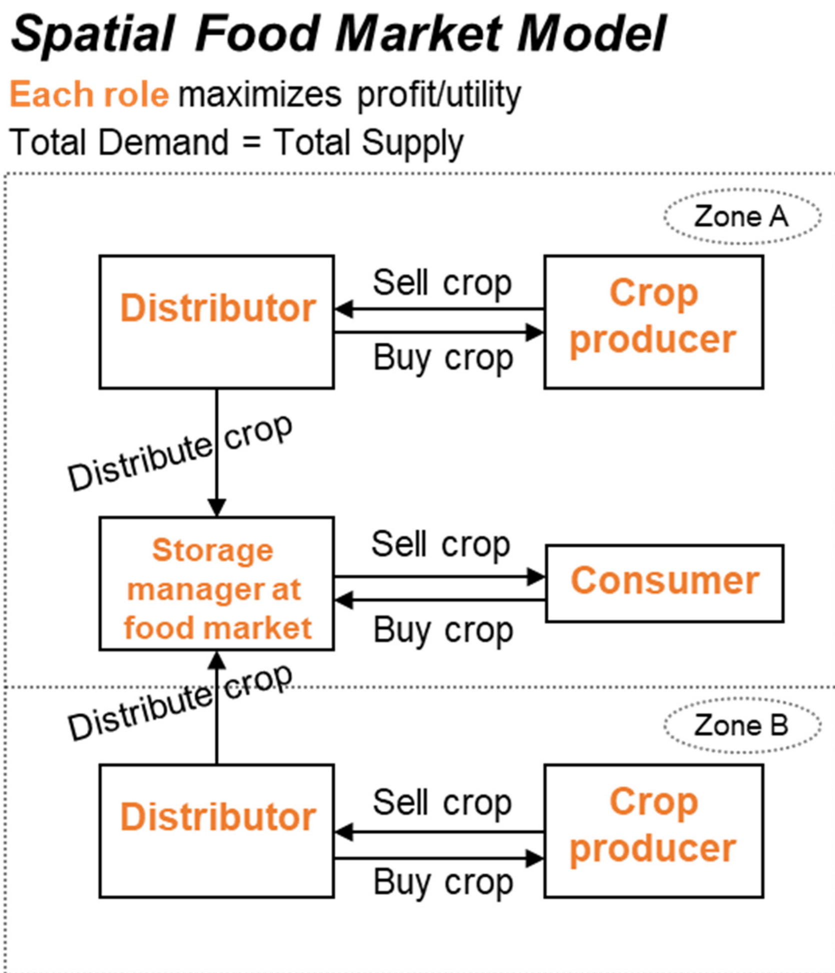

Fig. S2. Simplified illustration of the spatial food market model.

We calibrate the runoff coefficient γ to estimate Rl, and then partition Rl into the surface runoff Rs and groundwater recharge Rg. The calibration of γ is performed for each grid (i.e., each soil column) at daily basis given the simulation outputs from the Noah Multi-Parameterization Land Surface Model (Noah-MP LSM, version 3.6; Niu et al. 2011) within the framework of NASA's Land Information System (LIS; Kumar et al. 2006). The calibrated values of γ (average over all daily values in a grid) range from 0 to 10 with standard deviation ranging from 0 to 5 (Fig. S3).

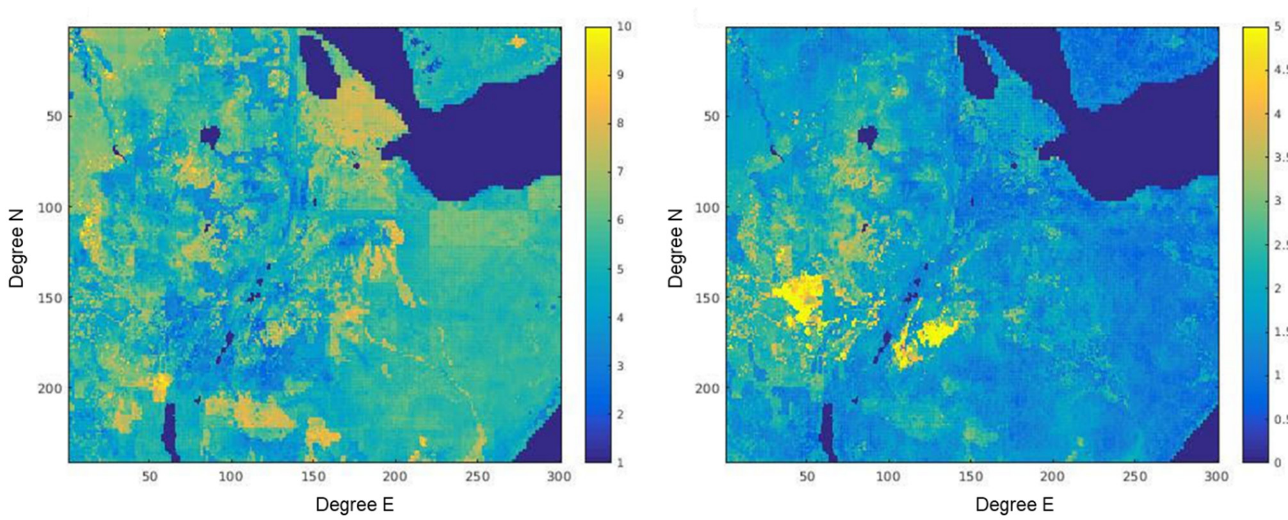

Fig. S3. Calibrated values of the runoff coefficient γ (left) and its standard deviation (right).

The partition of Rl into Rs and Rg is performed such that the resultant Rg converge with the calculated value of deep percolation (DP) in each grid. DP is calculated based on effective precipitation, soil water content, and crop evapotranspiration (Doorenbos et al. 1980; Allen et al. 1998).

The gridded yield factors for each crop estimated in the crop model are first aggregated to the level of adaptation zones, which is then taken as inputs to the spatial food market model DECO2. DECO2 then decides the area allocated to grow each crop in each adaptation zone. The resultant area allocation is used to calculate the groundwater storage change, considering irrigation, if any. The detailed coupling steps are described below [Eqs. (S1)–(S4) and Fig. S4].

$$\Delta S_{c,i} = S_{c,i} - S_0 \tag{S1}$$

where S_0 is the initial groundwater storage, which is assumed to be 5000mm. $S_{c,i}$ is the resultant groundwater storage at grid i as if crop c is grown at this grid, considering irrigation if any. Hence, $\Delta S_{c,i}$ is the corresponding groundwater change in the grid. This groundwater change is then multiplied by the ratio of the area allocated to the corresponding crop (an output from DECO2) over the total area of the adaptation zone that the grid belongs to. This calculation provides an estimate of the actual groundwater change resulting from the portion of the area allocated to grow that crop:

$$\Delta S_{c,i\in z}^{Real} = \Delta S_{c,i\in z} \times {A_{c,z}}/{Atot_z}$$

(S2)

where $Atot_z$ is the total area of adaptation zone z. $A_{c,z}$ is the area allocated to grow crop c in adaptation zone z.

$$\Delta S_{c,z}^{Real} = \sum_{i\in z} \Delta S_{c,i}^{Real}$$

(S3)

The gridded 'real' groundwater storage change is then aggregated to the level of adaptation zones. The effects on zonal groundwater storage change are also aggregated over all the modeled crops to obtain the overall groundwater change at each adaptation zone:

$$\Delta S_{z}^{Real} = \sum_{c} \Delta S_{c,z}^{Real}$$

(S4)

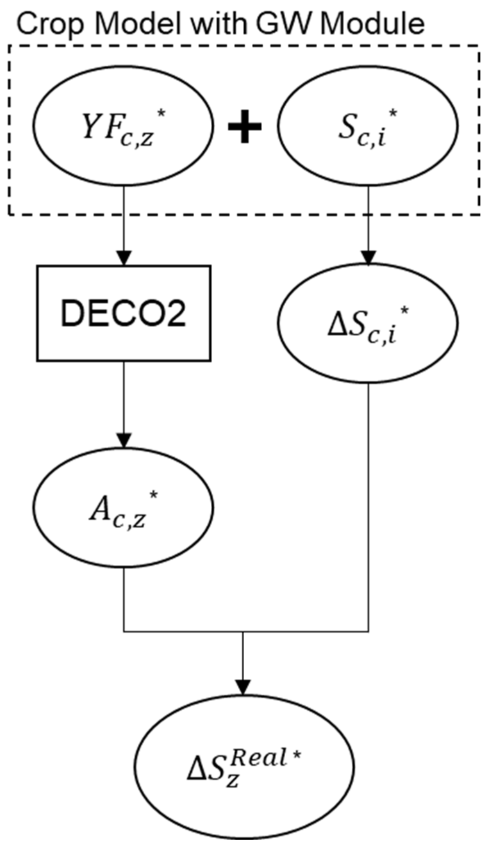

Fig. S4. Coupling of the crop model and DECO2 (the spatial food market model). $YF_{c,z}$ is yield factor for crop c at adaptation zone z. $S_{c,i}$ is the resultant groundwater storage at grid i as if crop c is grown at this grid, considering irrigation if any. $A_{c,z}$ is allocated area to grow crop c at adaptation zone z. ΔS_z^{Real} is the actual groundwater change considering the allocated crop patterns in adaptation zone z. The asteroid indicates that the variable would be updated when the coupled model runs under the irrigation scenario versus the baseline.

Table S1. Summary table of model parameters and variables

Symbol	Explanation	Symbol type
y	Year	Set
s	Season	Set
z	Adaptation zone	Set
n	Market	Set
c	Crop	Set
ξ	Animal produce – beef and milk	Set
f	Food (i.e., Crops and animal produce together)	Set
r	Roads (Connecting two markets s_r and d_r – the source and destination)	Set
R_{in}^n, R_{out}^n	Set of roads coming into and going out of a market location n	Set
df_y	Discount factor for year y	Parameter
Ψ_{zn}	Fraction of adaptation zone z that is in the Thiessen polygon corresponding to market n	Parameter
$Q_{znfsy}^{F\to D}$	Quantity of food f sold by producer in z to a distributor in market n in the season s of year y	Primal Variable
$\pi_{nfsy}^{D\to F}$	Price for food f paid by a distributor to any producer in market n in the season s of year y	Dual Variable
A_{zcsy}	Area allotted for crop c by the crop producer in z in the season s of year y	Primal Variable
Q_{zfsy}^F	Quantity of food f produced in adaptation zone z in the season s of year y	Primal Variable

C^{F}_{zcsy}	Cost of raising unit area of crop c for the crop producer in z in the season s of year y	Parameter
C^{chg}_{zsy}	Penalty for change in cropping patterns between adjacent years	Parameter
Y_{zcsy}	Yield of crop c in adaptation zone z in the season s of year y	Parameter
Q^{C}_{zsy}	Number of cattle reared in adaptation zone z in the season s of year y	Primal Variable
Q^{sl}_{zsy}	Number of cattle slaughtered in the adaptation zone z in the season s of year y	Primal Variable
$Q^{C,\xi}_{zsy}$	Quantity of animal product ξ produced in the adaptation zone z in the season s of year y	Primal Variable
$Q^{C\to D,\xi}_{znsy}$	Quantity of animal product ξ sold by the livestock producer in z to the distributor in market n in the season s of year y	Primal Variable
$\pi^{D\to C,\xi}_{nsy}$	Price for animal product ξ paid by the distributor in market n to any livestock producer in the season s of year y	Dual Variable
C^{C}_{zsy}	Cost of rearing cattle in the adaptation zone z in the season s of year y	Parameter
Y^{ξ}_{zsy}	Yield of animal product ξ in the adaptation zone z in the season s of year y	Parameter
k	Proportion of cattle that should be slaughtered before they die	Parameter
r_{birth}, r_{death}	Birth and death rate of cattle	Parameter
$Q^{C,Herd}_{zsy}$	Minimum heard size to be maintained	Parameter
$Q^{D_b}_{nfsy}$	Quantity of food f brought by distributor in market n in season s of year y	Primal Variable

$Q^{D_s}_{nfsy}$	Quantity of food f sold by distributor in market n in season s of year y	Primal Variable
Q^{D}_{rfsy}	Quantity of food f transported by distributor through road r in season s of year y	Primal Variable
C^{D}_{rfsy}	Cost of transporting food f through road r in season s of year y	Parameter
$\underline{Q^{D}_{rsy}}$	Maximum Quantity of food that can be transported through road r in season s of year y	Parameter
$\pi^{W\rightarrow D}_{nfsy}$	Price for food item f paid by a store to a distributor in market n in the season s of year y	Dual Variable
$\pi^{U\rightarrow W}_{nfsy}$	Price for food item f paid by a consumer to a store in market n in the season s of year y	Dual Variable
$Q^{W_b}_{nfsy}$	Quantity of food f brought by the store in market n in season s of year y	Primal Variable
$Q^{W_s}_{nfsy}$	Quantity of food f sold by the store in market n in season s of year y	Primal Variable
Q^{W}_{nfsy}	Quantity of food f stored by the store in n in season s of year y	Primal Variable
C^{W}_{nfsy}	Cost of storing food f at n in season s of year y	Parameter
$\underline{Q^{W}_{nfsy}}$	Maximum Quantity of food f that can be stored at n in season s of year y	Parameter
a_{zfsy}	Intercept of the demand curve for food f for consumers in adaptation zone z during season s of year y	Parameter
b_{zfsy}	Slope of the demand curve for food f for consumers in adaptation zone z during season s of year y	Parameter

Q^{U}_{zfsy}	Quantity of food f consumed in adaptation zone z during season s of year y	Primal variable
$Q^{W\to U}_{znfsy}$	Quantity of food f bought by consumers in adaptation zone z from the market n during season s of year y	Primal variable
$\underline{Q^{U}_{zfsy}}$	Consumption limit for food f in adaptation zone z during season s of year y (to model export)	Parameter

Mathematical Formulations of DECO2

1. Crop producer

$$\sum_{c,s,y} df_y \left(\sum_n Q^{F\to D}_{zncsy} \pi^{D\to F}_{ncsy} - C^F_{zcsy} A_{zcsy} \right) - \sum_{c,s,y,n} \left(\frac{1}{2} C^{chg}_{zsy} \left(A_{zcsy} - A_{zcs(y-1)} \right)^2 \right)$$

subject to

$$\begin{aligned}
Q^{F\to D}_{zncsy}, A_{zcsy}, Q^F_{zcsy} &\geq 0 && \\
\textstyle\sum_c A_{zcsy} &= A^{Total}_{zs} && (\delta^1_{zsy}) \\
Q^F_{zcsy} &= Y_{zcsy} A_{zcsy} && (\delta^2_{zcsy}) \\
Q^{F\to D}_{zncsy} &= \Psi_{zn} Q^F_{zcsy} && (\delta^{18}_{zncsy})
\end{aligned}$$

The KKT conditions corresponding to this optimization problem are as follows.

$$\begin{aligned}
\delta^2_{zcsy} - \textstyle\sum_n \Psi_{zn} \delta^{18}_{zncsy} &\geq 0 && (Q^F_{zcsy}) \\
\delta^1_{zsy} + df_y \left(C^F_{zcsy} + C^{chg}_{zsy} A_{zcsy} + C^{chg}_{zs(y+1)} A_{zcsy} \right) & && \\
- df_y \left(C^{chg}_{zsy} A_{zcs(y-1)} + C^{chg}_{zs(y+1)} A_{zcs(y+1)} \right) - &\geq 0 && (A_{zcsy}) \\
\delta^2_{zcsy} Y_{zcsy} & && \\
\delta^{18}_{zncsy} - df_y \pi^{D\to F}_{ncsy} &\geq 0 && (\delta^2_{zcsy})
\end{aligned}$$

2. Livestock producer

$$\sum_{s,y} df_y \left(\sum_{n,\xi} Q^{C\to D,\xi}_{znsy} \, \pi^{D\to C,\xi}_{nsy} - C^C_{zsy} Q^C_{zsy} \right)$$

subject to

$$
\begin{aligned}
Q^{sl}_{zsy}, Q^{C}_{zsy}, Q^{C,\xi}_{zsy} &\geq 0 \\
Q^{C}_{zsy} &= (1 + r_{birth} - r_{death})Q^{C}_{zs(y-1)} - Q^{sl}_{zs(y-1)} && (\pi^{C}_{zsy}) \\
Q^{sl}_{zsy} &\geq kQ^{C}_{zsy} && (\delta^{9}_{zsy}) \\
Q^{C}_{zsy} &\geq Q^{C,Herd}_{zsy} && (\delta^{10}_{zsy}) \\
Q^{C,milk}_{zsy} &= Y^{milk}_{zsy} Q^{C}_{zsy} && (\delta^{2}_{z(milk)sy}) \\
Q^{C,beef}_{zsy} &= Y^{beef}_{zsy} Q^{sl}_{zsy} && (\delta^{2}_{z(beef)sy}) \\
Q^{C\to D,\xi}_{znsy} &= \Psi_{zn} Q^{C,\xi}_{zsy} && (\delta^{18}_{zn\xi sy})
\end{aligned}
$$

The KKT conditions corresponding to this optimization problem are as follows.

$$
\begin{aligned}
df_y C^{C}_{zsy} - \delta^{2}_{z(milk)sy} Y^{(milk)}_{zsy} - \delta^{4}_{zsy} + kd^{9}_{zsy} - \\
d^{10}_{zsy} - (1 + r_{birth} - r_{death})\pi^{C}_{zsy} &\geq 0 && \left(Q^{C}_{zsy}\right) \\
\delta^{2}_{z\xi sy} - df_y \Psi_{zn} \delta^{18}_{zn\xi sy} &\geq 0 && (Q^{C,\xi}_{zsy}) \\
\delta^{4}_{zsy} - \delta^{9}_{zsy} - \delta^{2}_{z(beef)sy} Y^{beef}_{zsy} + \pi^{C}_{zs(y+1)} &\geq 0 && \left(Q^{sl}_{zsy}\right) \\
\delta^{18}_{zn\xi sy} - df_y \pi^{D\to C,\xi}_{nsy} &\geq 0 && \left(Q^{C\to D,\xi}_{znsy}\right)
\end{aligned}
$$

3. Distributor

$$
\sum_{f,s,y} df_y \left\{ \sum_{n} \left(Q^{D_s}_{nfsy} \pi^{W\to D}_{nfsy} - Q^{D_b}_{nfsy} \pi^{D\to F}_{nfsy} \right) - \sum_{r} C^{D}_{rfsy} Q^{D}_{rfsy} \right\}
$$

subject to

$$
\begin{aligned}
Q^{D_b}_{nfsy}, Q^{D_s}_{nfsy}, Q^{D}_{rfsy} &\geq 0 \\
\textstyle\sum_{r\in R^{n}_{in}} Q^{D}_{rfsy} + Q^{D_b}_{nfsy} &= \textstyle\sum_{r\in R^{n}_{out}} Q^{D}_{rfsy} + Q^{D_s}_{nfsy} && \left(\delta^{6}_{nfsy}\right) \\
\textstyle\sum_{f} Q^{D}_{rfsy} &\leq \underline{Q^{D}_{rsy}} && \left(\delta^{16}_{rsy}\right)
\end{aligned}
$$

The KKT conditions corresponding to this optimization problem are as follows.

$$df_y \pi^{D\to F}_{nfsy} - \delta^6_{nfsy} \geq 0 \qquad \left(Q^{D_b}_{nfsy}\right)$$

$$\delta^{16}_{rsy} + df_y C^D_{rfsy} + \delta^6_{s_r fsy} - \delta^6_{d_r fsy} \geq 0 \qquad \left(Q^D_{rfsy}\right)$$

$$\delta^6_{znfsy} - df_y \pi^{W\to D}_{nfsy} \geq 0 \qquad \left(Q^{D_s}_{nfsy}\right)$$

4. Storage manager (or stores)

$$\sum_{y,h,n,f} df_y \left(\pi^{U\to W}_{nfsy} Q^{W_s}_{nfsy} - \pi^{W\to D}_{nfsy} Q^{W_b}_{nfsy} - C^W_{nfsy} Q^W_{nfsy}\right)$$

subject to

$$Q^{W_b}_{nfsy}, Q^{W_s}_{nfsy}, Q^W_{nfsy} \geq 0$$

$$Q^W_{nf(sy-1)} + Q^{D_b}_{nfsy} = Q^W_{nfsy} + Q^{D_s}_{nfsy} \qquad \left(\delta^{11}_{nfsy}\right)$$

$$Q^W_{nfsy} \leq \underline{Q^W_{nfsy}} \qquad \left(\delta^8_{nfsy}\right)$$

The KKT conditions corresponding to this optimization problem are as follows.

$$df_y \pi^{U\to W}_{nfsy} - \delta^{11}_{nfsy} \geq 0 \qquad \left(Q^{W_b}_{nfsy}\right)$$

$$C^W_{nfsy} + \delta^8_{nfsy} + \delta^{11}_{nfsy} - \delta^{11}_{nf(sy+1)} \geq 0 \qquad \left(Q^D_{rfsy}\right)$$

$$\delta^{11}_{znfsy} - df_y \pi^{U\to W}_{nfsy} \geq 0 \qquad \left(Q^{W_s}_{nfsy}\right)$$

Here $sy - 1$ is the previous season (which could be in the previous year) and $sy + 1$ is the next season (which could be in the next year).

5. Consumer

$$\sum_{f,s,y} df_y \left(a_{zfsy} Q^U_{zfsy} - \frac{1}{2} b_{zfsy} {Q^U_{zfsy}}^2 - \sum_n \pi^{U\to W}_{nfsy} Q^{W\to U}_{znfsy} \right)$$

subject to

$$Q^{U}_{zfsy}, Q^{W\to U}_{znfsy} \geq 0$$

$$\sum_n Q^{W\to U}_{znfsy} = Q^{U}_{zfsy} \qquad (\delta^{19}_{zfsy})$$

$$Q^{W\to U}_{znfsy} \geq \Psi_{zn} Q^{U}_{zfsy} \qquad (\delta^{17}_{znfsy})$$

$$Q^{U}_{zfsy} \leq \underline{Q^{U}_{zfsy}} \qquad (\delta^{20}_{zfsy})$$

The KKT conditions corresponding to this optimization problem are as follows.

$$df_y\left(b_{zfsy} Q^{U}_{zfsy} - a_{zfsy}\right) + \delta^{18}_{zfsy} + \delta^{20}_{zfsy} + \sum_n \Psi_{zn} \delta^{17}_{znfsy} \geq 0 \qquad (Q^{W\to U}_{znfsy})$$

$$df_y \pi^{U\to W}_{nfsy} - \delta^{19}_{zfsy} - \delta^{17}_{znfsy} \geq 0 \qquad (Q^{U}_{zfsy})$$

6. Market clearing

$$\sum_z Q^{F\to D}_{znfsy} = Q^{D_b}_{nfsy} \qquad (\pi^{D\to F}_{nfsy})$$

$$Q^{D_s}_{nfsy} = Q^{W_b}_{nfsy} \qquad (\pi^{W\to D}_{nfsy})$$

$$Q^{W_s}_{nfsy} = \sum_z Q^{W\to U}_{znfsy} \qquad (\pi^{U\to W}_{nfsy})$$

References

ALLEN, R. G., PEREIRA, L. S., RAES, D. & SMITH, M. 1998. Crop evapotranspiration-Guidelines for computing crop water requirements-FAO Irrigation and drainage paper 56. *Fao, Rome,* 300**,** D05109.

DOORENBOS, J., KASSAM, A. H., BENTVELSEN, C. & UITTENBOGAARD, G. 1980. Yield Response to Water11This paper is a summary of: Doorenbos, J., A. H. Kassam, C. Bentvelsen, V. Branscheid, M. Smith, J . Plusje, G. Uittenbogaard and H. van der Wal; Yield response to water, FAO Irrigation and Drainage Paper, (In press). *In:* JOHL, S. S. (ed.) *Irrigation and Agricultural Development.* Pergamon.

KUMAR, S. V., PETERS-LIDARD, C. D., TIAN, Y., HOUSER, P. R., GEIGER, J., OLDEN, S., LIGHTY, L., EASTMAN, J. L., DOTY, B., DIRMEYER, P., ADAMS, J., MITCHELL, K., WOOD, E. F. & SHEFFIELD, J. 2006. Land information system: An interoperable framework for high resolution land surface modeling. *Environmental Modelling & Software,* 21**,** 1402-1415.

NIU, G.-Y., YANG, Z.-L., MITCHELL, K. E., CHEN, F., EK, M. B., BARLAGE, M., KUMAR, A., MANNING, K., NIYOGI, D., ROSERO, E., TEWARI, M. & XIA, Y. 2011. The community Noah land surface model with multiparameterization options (Noah-MP): 1. Model description and evaluation with local-scale measurements. *Journal of Geophysical Research: Atmospheres,* 116.

SANKARANARAYANAN, S., ZHANG, Y., CARNEY, J., NIGUSSIE, Y., ESAYAS, B., SIMANE, B., ZAITCHIK, B. & SIDDIQUI, S. 2020. What Are the Domestic and Regional Impacts From Ethiopia's Policy on the Export Ban of Teff? *Frontiers in Sustainable Food Systems,* 4.