SUPPLEMENTAL MATERIALS

ASCE *Journal of Environmental Engineering*

Quantifying Mixing in Sewer Networks for Source Localization

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DOI: 10.1061/JOEEDU.EEENG-7134

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Appendix S1. Calibration of fluorometers

Rhodamine WT fluorescence is temperature-dependent, and thus a temperature and concentration combined calibration was carried out after the experiments had been completed. The instruments and calibration solutions of 0, 250, 500, 750, and 1000 ppb (made using distilled water) were placed in a climate-controlled chamber at the Arthur Willis Environment Centre (The University of Sheffield, UK) programmed to remain at 5, 10, 15, and 20 °C each for 24 hours. After at least 16 hours to allow temperatures to reach steady-state, each instrument was placed in each calibration solution for five to ten minutes. The standard logarithmic fluorescence temperature correction formula is $c_s = c\exp(n_t(T_s - T))$, where c_s is the concentration at a standard temperature, c is the measured concentration, n_t is a temperature exponent, T_s is a standard temperature, and T is the measured temperature (Smart and Laidlaw, 1977). The concentration readings and readings from the logger's built-in temperature sensor were fit with least-squares optimisation to the correction formula to find a temperature exponent of $n_t = -0.0227\ °C^{-1}$. After correction to a standard temperature of 20 °C, a linear calibration equation was obtained for each instrument, shown in Fig. S1. The calibrations showed good linearity with $R_t^2 > 0.999$. Unfortunately, it was not possible to perform a full calibration both before and after the experiments. Comparison to an earlier partial calibration suggests drift on the order of 1%/year.

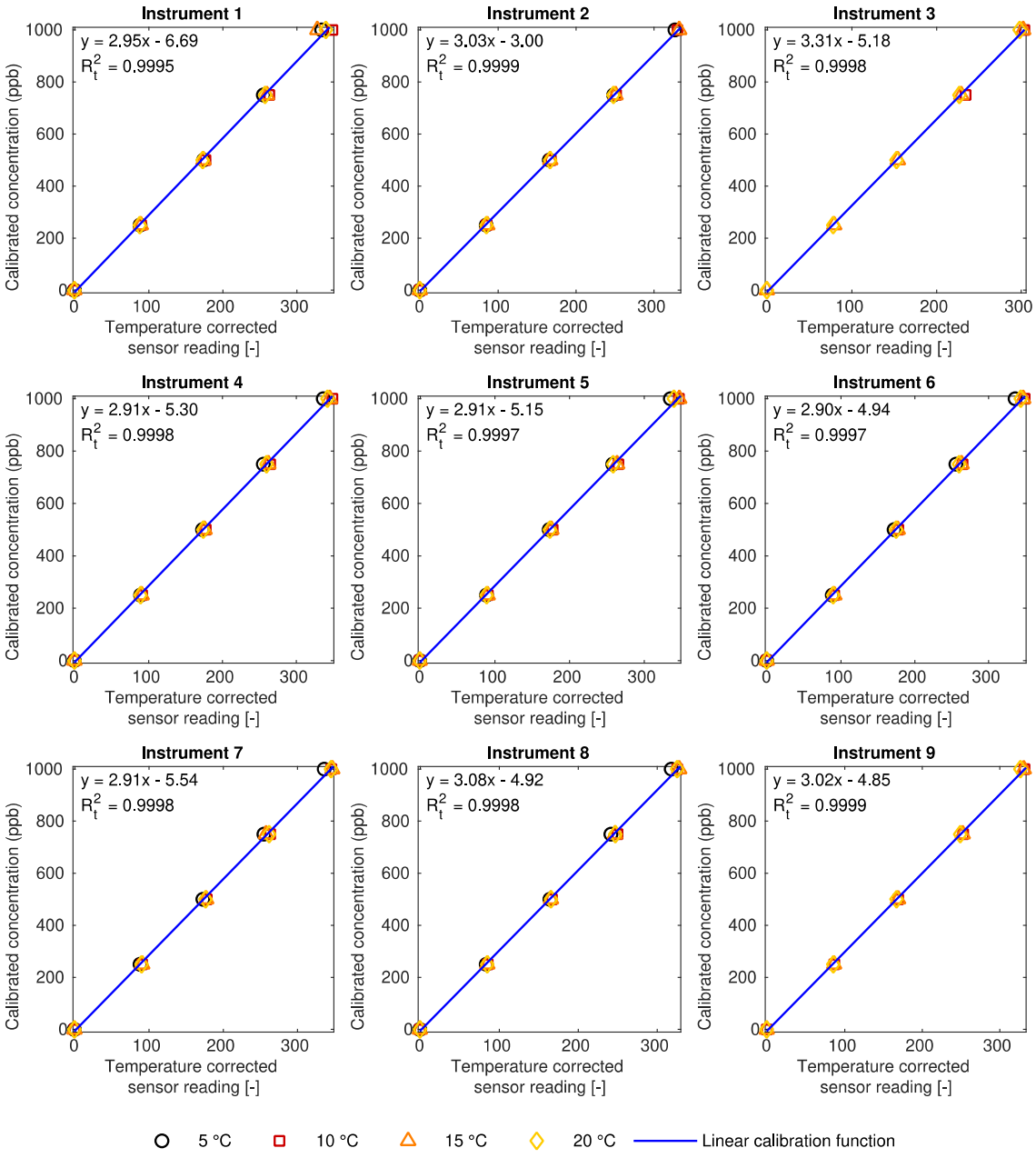

Fig. S1. Temperature corrected fluorometer calibrations.

Appendix S2. Description of simplified hydraulic and solute transport model

Assuming steady-state and a best practice dry weather flow channel (WRC, 2012), sewer flow can be treated as an open channel with a circular segment cross-section. Knowing discharge, Manning's equation can be applied to estimate velocity, and hence, travel time:

$$\frac{Q}{A} = U = \frac{1}{n} R^{2/3} S_0^{1/2} \tag{S1}$$

where A is the flow's cross-sectional area, n is Manning's roughness coefficient, R is the hydraulic radius (cross-sectional area divided by wetted perimeter), and S_0 is channel slope (Chaudry, 2008). Camp (1946) showed that n varied in a partially full pipe compared to a pipe flowing full, and this has been taken into consideration using a piecewise function (Bengtson, 2012).

Solute transport routing (Eq. 2) can be considered a simple steady-state solute transport model, for which only D_x must be estimated as U and $\bar{t}$ are provided by the hydraulic component of the model. Equation 2 assumes a Gaussian transfer function relating upstream and downstream locations, i.e., Fickian dispersion. Although slightly non-Gaussian solute transport behaviour does occur in sewers (Rieckermann et al., 2005; Sokáč and Velísková, 2016), Eq. 2 is applicable after a sufficiently long distance after injection (Fischer et al., 1979). As Eq. 2 is analytical, the proposed solute transport model is not susceptible to numerical dispersion unlike commercial modelling packages (Bouteligier et al., 2005).

A simple network model is constructed when the hydraulics and solute transport models for each pipe are connected, which for convenience can be considered a many-to-one network graph. In such a graph, each manhole is a node that may be connected upstream to one or more manholes (through pipes or conduits) but may not connect to more than one downstream manhole, e.g., Fig. 2. At each manhole, the outflow is the sum of inflows plus any baseflow, and concentrations are mass-balanced according to the ratio of flows. While bifurcations may exist in practice, these have been ignored for simplicity.

The simplified model described here has been implemented using a hybrid of MATLAB (MathWorks Inc., 2022) and Python (Van Rossum, 2009). When only equivalent sand-grain roughness k_s was available, n was calculated as $n = k_s^{1/6}/\left(8.25\sqrt{g}\right)$ where g is the acceleration due to gravity (Chaudry, 2008). Model inputs to the hydraulic portion of the model are network geometry and flow rates, and outputs are flow depth, velocity, hydraulic radius, and surface width. The inputs to the solute transport portion of the model, in addition to the hydraulic portion outputs, are an upstream concentration distribution and a dispersion coefficient function (e.g., Eq. 6) to predict a downstream concentration distribution as the output.

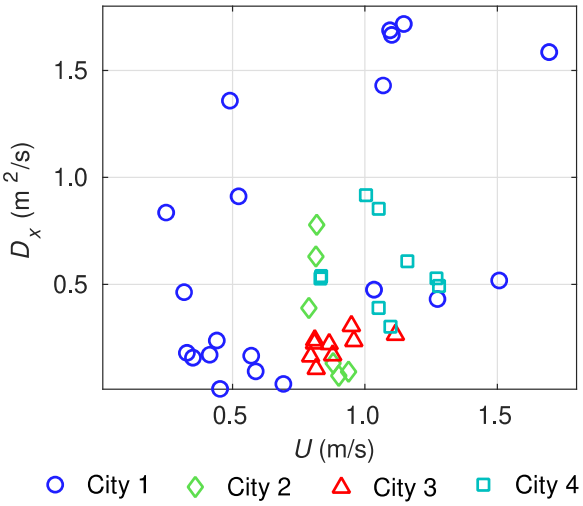

Fig. S2. Velocity and longitudinal dispersion coefficients optimised from the recorded traces.

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