SUPPLEMENTAL MATERIALS

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Improved Prediction of Managed Water Flow into Everglades National Park Using Empirical Dynamic Modeling

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S1. Convergent cross mapping (CCM (methods from Sugihara, May et al. 2012) to assess causal drivers of flow targets. CCM measures how well variable x maps onto variable y. In short, if x can be used to predict values of y, that means y is a causal driver on x. Cross map skill is measured by Pearson's correlation (ρ) between observed and predicted values. "Convergence" refers to how mapping accuracy improves as more data is included in state-space reconstructions (library size). If a relationship is state dependent (nonlinear), more data is needed to accurately define distinct states, thus improving predictions with larger library sizes. It is also instructive to compare the converged value of CCM to the linear absolute cross-correlation (red horizontal lines) between variables. If the correlation obtained with CCM is less than the linear cross-correlation, the relationship is likely not nonlinear; rather, potentially strictly linear or synchronized. Here, we use CCM to map from Q^{sum} onto each potential driving variable, thus measuring each variables' influence on flow targets. We find that all variables show convergence to values greater than the linear cross-correlation, suggesting nonlinear coupling between flow targets and each variable. However, the cross-map skill is highly variable across the drivers (bottom right panel). The fact that Q^{sum} maps much more accurately onto the NESRS and WCA3A water levels suggests that these are the strongest drivers. ZA seems to have the next strongest mapping; however, this is likely largely due to the strictly seasonal periodicity of this variable. PET and Rain have the weakest CCM values, suggesting they are the weakest drivers of flow targets. Interestingly, mapping from Q^{sum} onto the water level in the NESRS region converges to a value significantly greater (ρ_{CCM} = 0.93) than that of the linear correlation (ρ_{Cor} = 0.74), suggesting that the relationship between NESRS and Q^{sum} is very strong and highly state-dependent.

S2. Multiview embedding (Ye and Sugihara 2016) is an EDM algorithm that seeks to identify an optimal state-space from combinations of state variables. This differs from the reconstructions done in the S-Map analysis shown in figure 7 in that multiview allows for multiple delays of each variable, permitting many combinations of multivariate, time-lagged embeddings. It then searches D-dimensional state-spaces constructed from combinations of these state variables assessing the model predictability (information content) of each state-space. Here we chose D, the dimensionality of the state-space reconstruction, based on an assessment of the optimal univariate embedding (see methods of Sugihara & May 1990) of the target variable. Using these multivariate embeddings with E = 5, and D = 2, 3, 4, we predicted $Q^{sum}(t)$ and evaluated state-spaces. These values were chosen based on an assessment of optimal embedding dimension of E = 3 for $Q^{sum}(t)$. Here, we show the variables used in each optimal embeddings across multiple sets of parameters: a) Full record, in-sample predictions using a D = 3-dimensional state-space. b) Out-of-sample predictions with 2-D state-space. c) Out-of-sample predictions with 3-D state-space. d) Out-of-sample predictions with 4-D state-space. As an example, c) shows that for a D = 3-dimensional state-space constructed from E = 5-dimensional time-delay embeddings of the five variables: WCA-3A, NESRS, Rain, PET, ZA; that, of the best predicting N = 47 state-spaces, the first variable is always selected as

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WCA-3A with a time delay of 0 or 1 week. The second variable is overwhelmingly selected as NESRS, and ZA for the third variable. Collectively, these results indicate that water levels in WCA-3A and NESRS are the most informative variables for prediction of $Q^{sum}(t)$

S3. Different sized windows for computing regressive fits obtain differing results. In the window sizes tested, 5-year gives relatively the least amount of error in predictions.