

# A Hybrid Method for Calculating Residual Stress and Deformation in Space Hardware on Earth

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## Abstract

Many times in testing space hardware it is desirable to quantify residual von Mises stresses and deformation in a test article which will ultimately be placed in a micro-gravity environment. When testing in a 1-g environment the stresses and deformations include the contributions due to weight. This investigation demonstrates a hybrid method used to identify the residual von Mises and deformations that reflect a no weight condition. To this end, an example of a hard X-ray space mirror is presented to demonstrate this hybrid method which combines finite element analysis with experimental profile measurements, made under gravity loading, on a prototype formed by electroforming a thin shell on a cylindrical aluminum mandrel.

## Keywords

Residual stress, finite element analysis, hybrid method

## Introduction

To illustrate a hybrid method of calculating residual stresses in an article of space hardware an example of a hard x-ray mirror is considered. Direct determination of residual stress can be made with several methods, for instance, indentation (Suresh and Giannakopoulos 1998) but can be costly and time-consuming. However, a simple and less expensive approach may be used for early design purposes. An example of this procedure, using an x-ray mirror is described below.

41 . The Constellation-X Observatory was a mission concept for an X-ray space observatory to be operated  
42 by NASA. The objective was to investigate black holes, Einstein's Theory of General Relativity, galaxy formation,  
43 the evolution of the Universe on the largest scales, the recycling of matter and energy, and the nature of "dark  
44 matter."

45 The telescope aboard the satellite was designed to operate in the ~40 keV portion of the electromagnetic  
46 spectrum and, as illustrated in Fig. 1, relied on an array of primary mirrors nested within each other. The mirrors  
47 follow mathematical curves - the parabola and hyperbola - derived by slicing through an imaginary cone at  
48 different angles.

49 X-ray telescopes are very different from optical telescopes, because with their high energies, X-ray  
50 photons will simply pass through a conventional mirror. To solve this problem, the mirrors are cylindrically-  
51 shaped so that hard X-rays (~ 40 keV) are deflected into the instrument like stones skipping off water.

52 In a Wolter-I design (Wolter 1952), incoming photons undergo two reflections, the first from a parabolic  
53 surface and the second from a hyperbolic surface, to give an image that is essentially coma free. One approach  
54 developed to construct this design is to produce full-shell, shallow-graze-angle, gold-coated replicated mirrors by  
55 using electroformed nickel replication (Ramsey et al 1999, Ramsey et al 2000).

56 The advantage of electroforming is that complex coating procedures are avoided. The process lends  
57 itself readily to the multiple-mirror-module approach that small graze angles necessitate and the resulting shells  
58 provide excellent angular resolution that results in high sensitivity observations. This in turn translates directly  
59 into greater sensitivity through reduced focal spot size. Finally, with the use of high strength alloys one can  
60 achieve the stringent weight requirements of space-based missions.

61 During the mirror fabrication process, nickel mirror shells are electroformed onto a figured and highly  
62 polished aluminum mandrel from which they are later released by differential thermal contraction. Figure 2  
63 illustrates that the resulting mirror shells are full circles of revolution.

64 The axisymmetric geometry provides good structural stability permitting good figure accuracy, and hence  
65 very good angular resolution. Since nickel has a high density, researchers must make very thin shells to achieve  
66 the lightweight optics necessary to keep launch costs reasonable.

67 The shells must be strong enough to withstand the stresses of fabrication and subsequent handling  
68 without undergoing permanent deformation. They must also be electroformed in an ultra-low-stress environment

69 to prevent stress-induced distortions once they are released. In short, the challenge is to maintain high angular  
70 resolution despite small weight-budget-driven mirror shell thickness. These requirements make shells extremely  
71 sensitive to the fabrication process and handling stresses.

72 The current mandrels used for electroforming represent conical approximations to Wolter-I geometry and  
73 typical metrology gives a performance prediction for the shells of around 8- to 10-arcsec half-power diameter  
74 (HPD), meaning half the reflected flux from a point source falls within this angular range. However, X-ray tests  
75 reveal shell performances in the 13- to 15-arcsec range with modules running around 17-arcsec HPD (Ramsey et  
76 al 2000, Ramsey et al 2002). Consequently, it is essential to identify the source and extent of these  
77 discrepancies so that steps can be taken to correct them.

78 In a recent study (Franco 2003), laser techniques were applied to profile an electroplated shell after it was  
79 manufactured. When the shell was positioned vertically on a flat surface under laboratory conditions, profile  
80 measurements revealed that distortions in the mirrored surface created excessive optical distortion in the  
81 telescopic system. It was hypothesized that anomalies in the shape of the shell were due to residual stresses  
82 developed either during the electroplating process or while the shell was thermally removed from its mandrel.  
83 The removal process was eliminated as a source for these anomalies by further research (Franco et al 2005).

#### 84 **Hybrid Analysis**

85 The hybrid approach consists of determining the von Mises stress due to electroplating by subtracting the  
86 stress on the shell created by a gravity loading from the total stress computed based on profile measurements.  
87 Simply put,

$$88 \quad \sigma_{\text{RESIDUAL}} = \sigma_{\text{EXPERIMENTAL}} - \sigma_{\text{GRAVITY}} \quad (1)$$

89 The residual deformations are obtained in a similar manner by subtracting the deformation due to gravity  
90 from the deformation computed based on profile measurements. Again,

$$91 \quad \delta_{\text{RESIDUAL}} = \delta_{\text{EXPERIMENTAL}} - \delta_{\text{GRAVITY}} \quad (2)$$

## 92 Conical Shell Manufacturing Process

93 Figure 3 shows the test article developed for this study. As illustrated in the schematic on the left, the  
94 mirror is 58.42 cm (23 in.) long and consists of an ultra thin shell that is slightly conical in shape. One half of the  
95 test article has a parabolic shape and an outer edge radius of 24.69 cm (9.72 in.) whereas the other half is  
96 hyperbolic with an edge radius of 24.66 cm (9.707 in.). Since the inner surface serves as a mirror for high  
97 resolution optical imaging, the shape of the shell must be carefully controlled during the manufacturing process.  
98 A cross section of the five layers of interest and their thicknesses for the case considered are shown to the right in  
99 Fig. 3.

100 The production of the mirror involves several steps. The first step is the fabrication of an aluminum  
101 mandrel with a radius of 0.102 mm (0.004 in.) below that required for the shell. Next, the mandrel is coated with  
102 0.099 mm (0.0039 in.) of electroless nickel to give a hard surface suitable for polishing. Then the mandrel is  
103 accurately figured using a cylindrical grinding machine. Finally, a mechanical super polishing takes place,  
104 sufficient to ensure that scattering does not dominate the mirror's performance up to the cut-off energy.

105 To prepare for electroforming, the surface of the mandrel is treated to form an oxide layer from which the  
106 shell can be easily released. Then the mandrel is immersed in the plating tank. A typical shell takes  
107 approximately 1 day to electroform, at which time the plated mandrel is taken from the bath, rinsed, and dried.  
108 Then the assembly is cooled to separate the shell from the mandrel. This is accomplished by immersing it into a  
109 dewar of liquid nitrogen and then sliding the mandrel from the shell as release takes place. The process relies on  
110 the differences in the thermal coefficients of expansion of the materials and the relative bond strength between  
111 the layers to separate the top two layers from the mandrel. The final configuration consists of an ultra thin, open  
112 ended, conical shell, with a gold mirror on the inside and a cobalt-nickel substrate on the outside.

## 113 Experimental Measurements

114 A series of experimental tests were conducted to support this work including thickness measurement,  
115 profiling, and yield stress determination. The shell was positioned vertically and profile measurements were taken  
116 on the outer surfaces along four meridians: 0, 90, 180, and 270 degrees. An average thickness of 0.333 mm

117 ( 0.0131 in.) was used in the finite element model to take into account the variations in thickness over the height.  
118 Thickness values ranged from 0.33 mm (0.0130 in.) to 0.34 mm (0.0134 in.).

119 The shell profiles were measured in two separate runs (Gubarev et al 2001). The parabolic section was  
120 profiled first. Then the shell was inverted and the hyperbolic section profiled.

121 Figures 4 and 5 show the deviation of each of the meridians from the design geometry in the parabolic  
122 (top) and hyperbolic (bottom) sections, respectively. In each case, the shell was supported at a height of zero  
123 [0.0 mm (0.0 in.)]. If the shell contour were perfect, the curves would fall along the abscissa. However, the plots  
124 show that the surface bulges outward along all of the meridians.

125 In the case of the parabolic section, a maximum deformation equal to 0.0089 mm ( $352 \times 10^{-6}$  in.) occurs  
126 at 10.24 cm ( 4.03 in.) from the bottom in the 0 deg meridian. Three local maxima are observed on the curves  
127 and the deformation decreases to zero at a height of 29.21 cm (11.5 in.), corresponding to the mid-section of the  
128 shell.

129 In the case of hyperbolic section, the shell bulges outward and a maximum deformation equal to .0067  
130 mm (262  $\mu$  in.) occurs at 1.57 mm (0.62 in.) from the bottom in the 180 deg meridian. Six local maxima are  
131 observed and the deformation decreases to zero at the mid-section, corresponding to a height of 29.21 cm (11.5  
132 in.).

### 133 **Finite Element Analysis**

134 The finite element model (FEM) used to analyze the shell was generated by using MSC/Nastran. See  
135 Fig. 6. Since the mirror is a shell, the model was meshed using 4-node quadrilateral (CQUAD4) elements. A total  
136 of 18480 elements and 18720 nodes were generated. The cobalt-nickel substrate was assumed to be linearly  
137 elastic, homogeneous, and isotropic with properties obtained from the tensile test. A convergence test consisting  
138 of doubling the mesh density to ascertain any change in results was conducted. It was concluded that the  
139 existing mesh was adequate.

140 Two load cases were analyzed while assuming that a gravity load was applied: the first with the average  
141 readings taken during profiling imposed along the meridians; the second with the shell subjected to gravity only.

142 In both cases the shell was assumed to be supported along the bottom edge at 12 equally spaced points. The  
143 tangential and longitudinal components were constrained at the supports while the radial and all rotation  
144 components remained free.

## 145 **Hybrid Results**

146 Equations 1 and 2 were applied to obtain the von Mises stress and radial deformation under microgravity  
147 conditions. Figure 7 shows the residual stress plotted along a meridian for the parabolic (left) and hyperbolic  
148 (right) sections. Figure 8 shows the results obtained for the residual radial deformation.

149 The stress and deformation are radially symmetric and, as illustrated in Figs. 9, 10 and 11, can be  
150 depicted three-dimensionally in the form of Patran plots. The maximum stresses and deformations for the two  
151 sections of the model are summarized in Table 1.

## 152 **Discussion**

153 As mentioned previously, any deviations from the desired profile (see Fig. 7) that result from unwanted  
154 residual stress (see Fig. 8) produce aberrations that reduce the overall optical performance of the device.  
155 Knowing the magnitude and distribution of these quantities will help designers make improvements.

156 A stress approach could be taken where the shell was purposefully deformed after fabrication to induce a  
157 permanent deformation equal and opposite to the deviation. However, this would require the shell to be ductile;  
158 unfortunately, this is not the case at present. An alternate approach would be to reconfigure the mandrel by  
159 undercutting it to account for the unwanted deformation.

160

## 161 **Conclusion**

162 The residual stress and deformation of a hard X-ray mirror under microgravity conditions can be  
163 determined with the use of a hybrid method. By subtracting the results obtained from a 1-g gravity loading from  
164 those derived experimentally from imposed deformation measured under laboratory conditions, the residual stress

165 and deformation from the manufacturing process can be quantified. This technique could be used in a myriad of  
166 space test articles and satellites. Further work needs to be done, specifically the deformation of a test article in  
167 an actual 0-g environment needs to be ascertained and compared to the theoretical predictions.

168 **Acknowledgement**

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Section	von Mises Stress	Radial Deformation
Hyperbolic	4.56 MPa (662 psi)	0.0065 mm ( $2.54 \times 10^{-4}$ in.)
Parabolic	5.32 MPa (772 psi)	0.0091 mm ( $3.58 \times 10^{-4}$ in.)

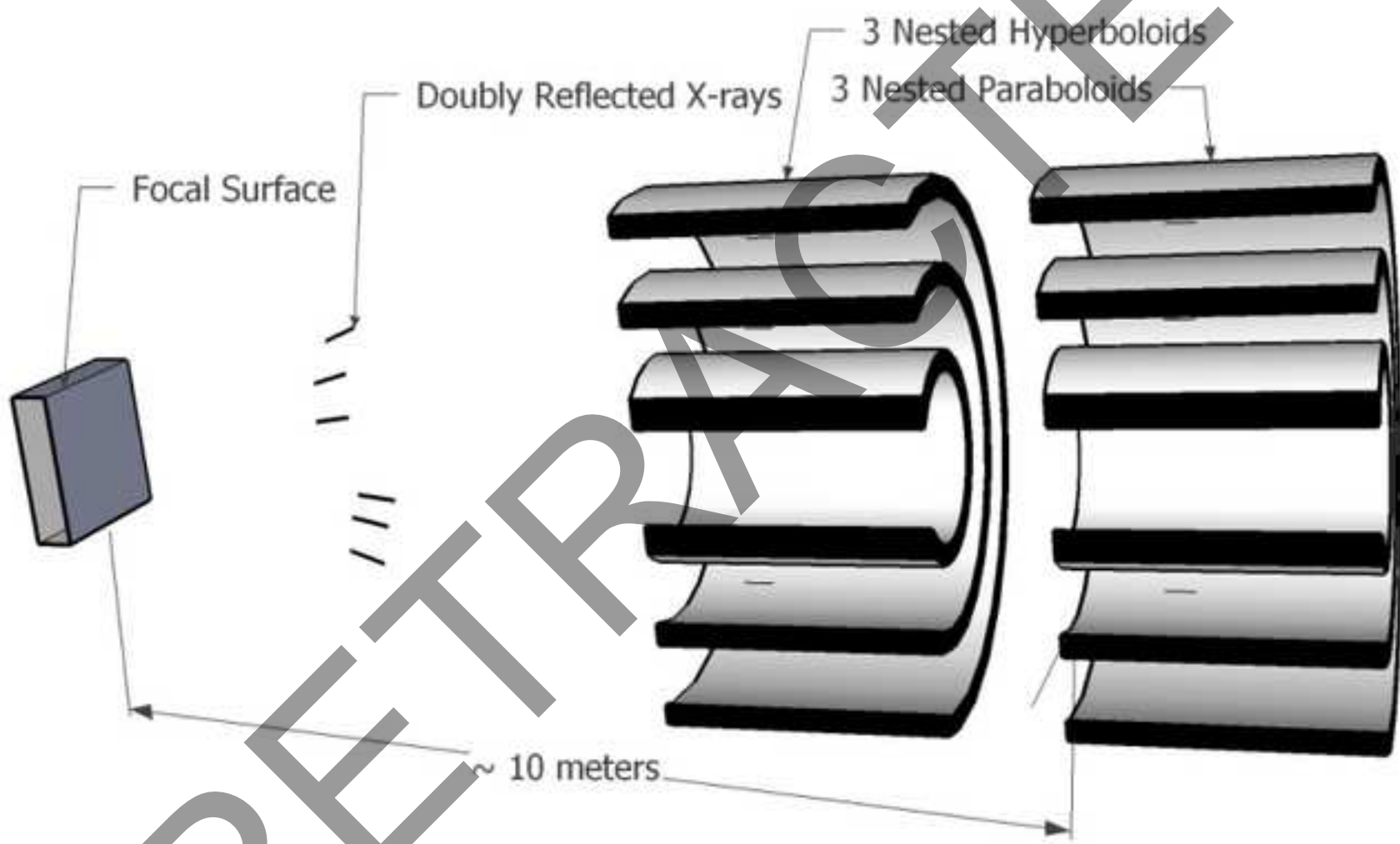
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**Table 1** FEM results for maximum residual stress and radial deformation

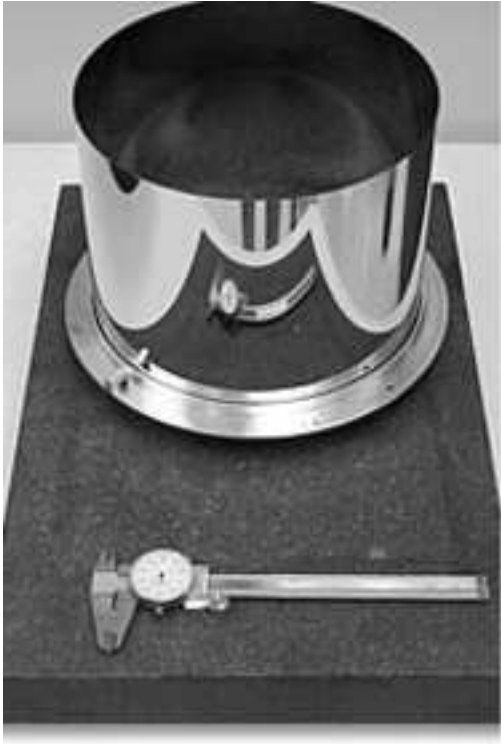
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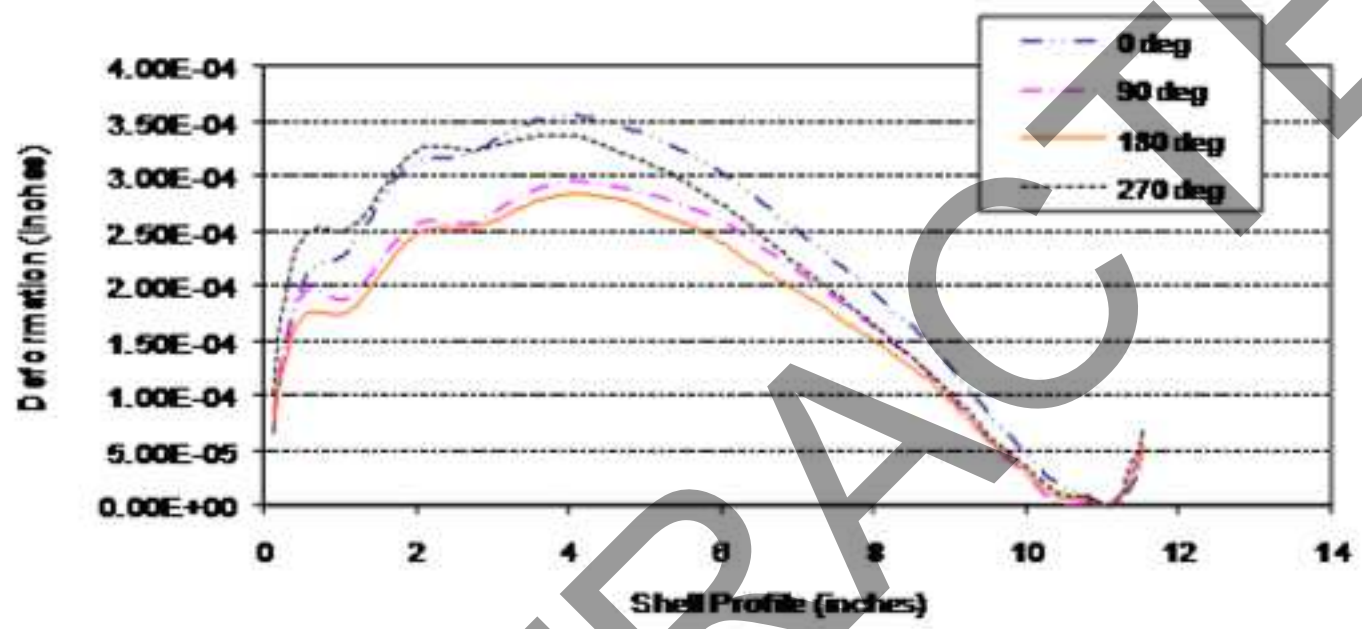


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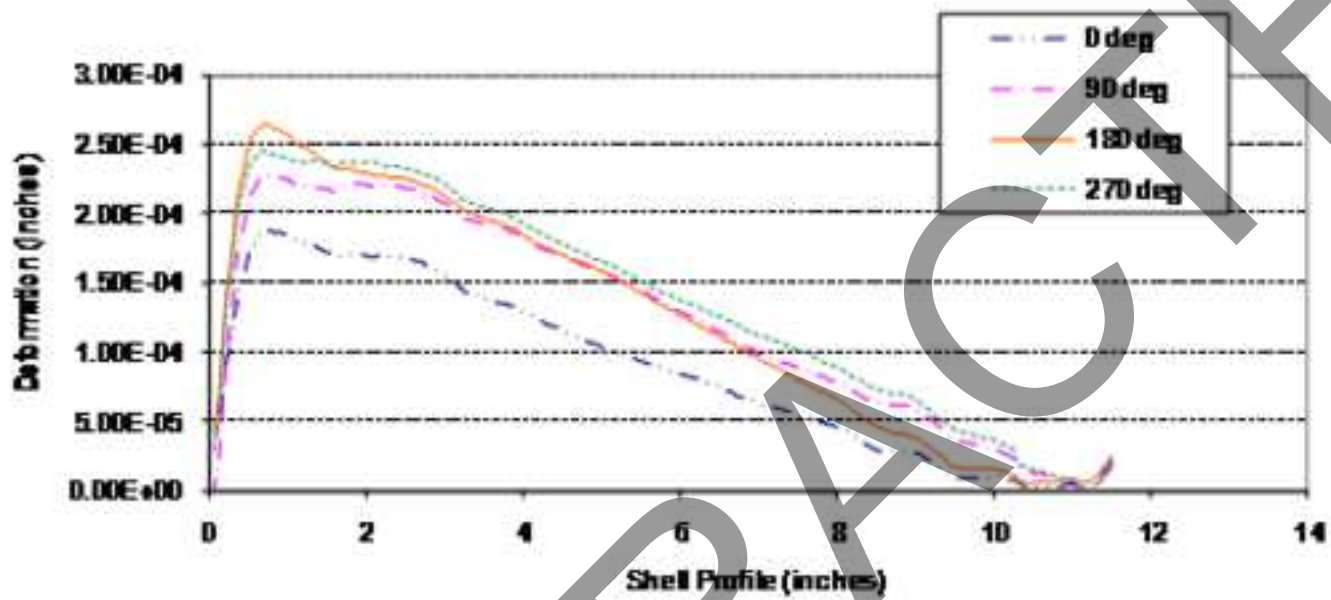


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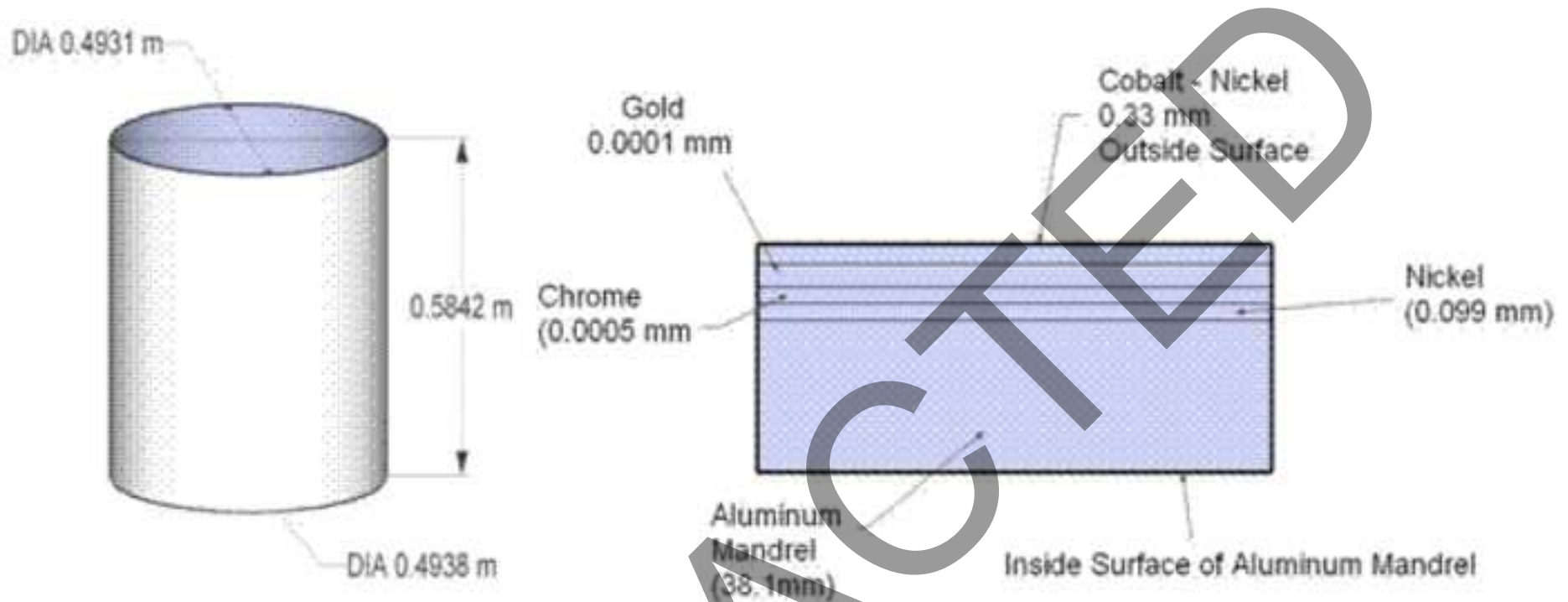


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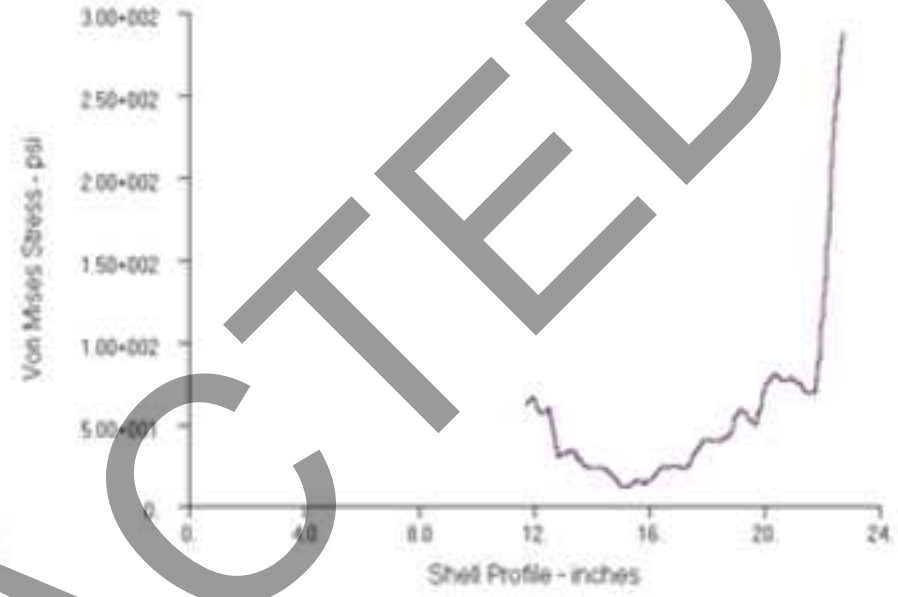
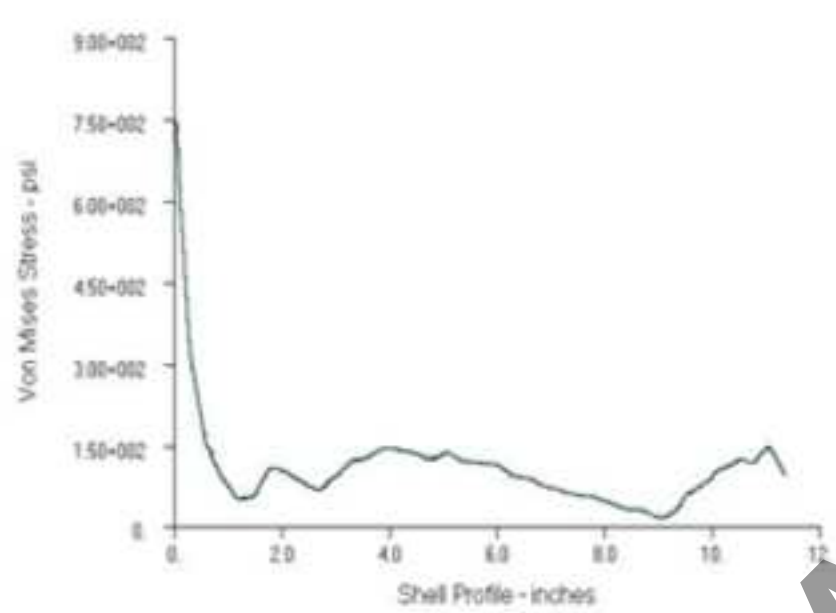


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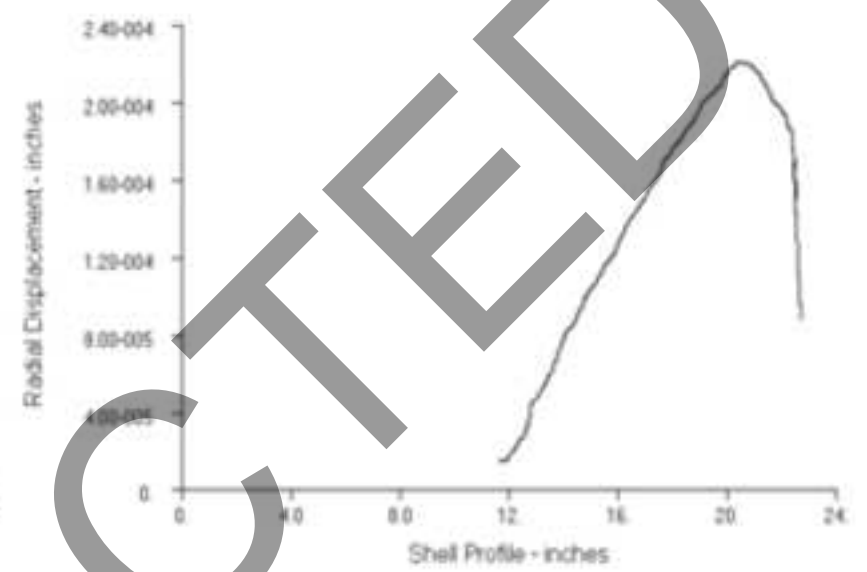
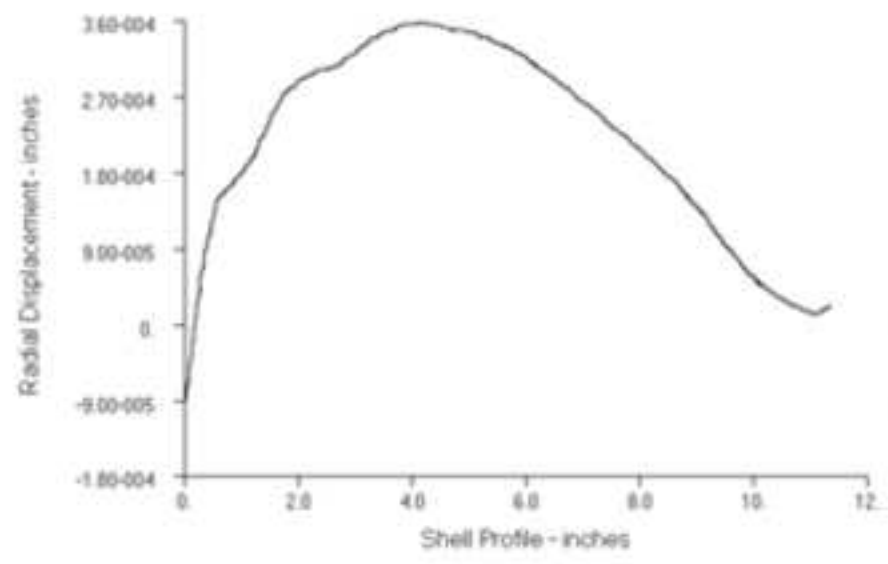
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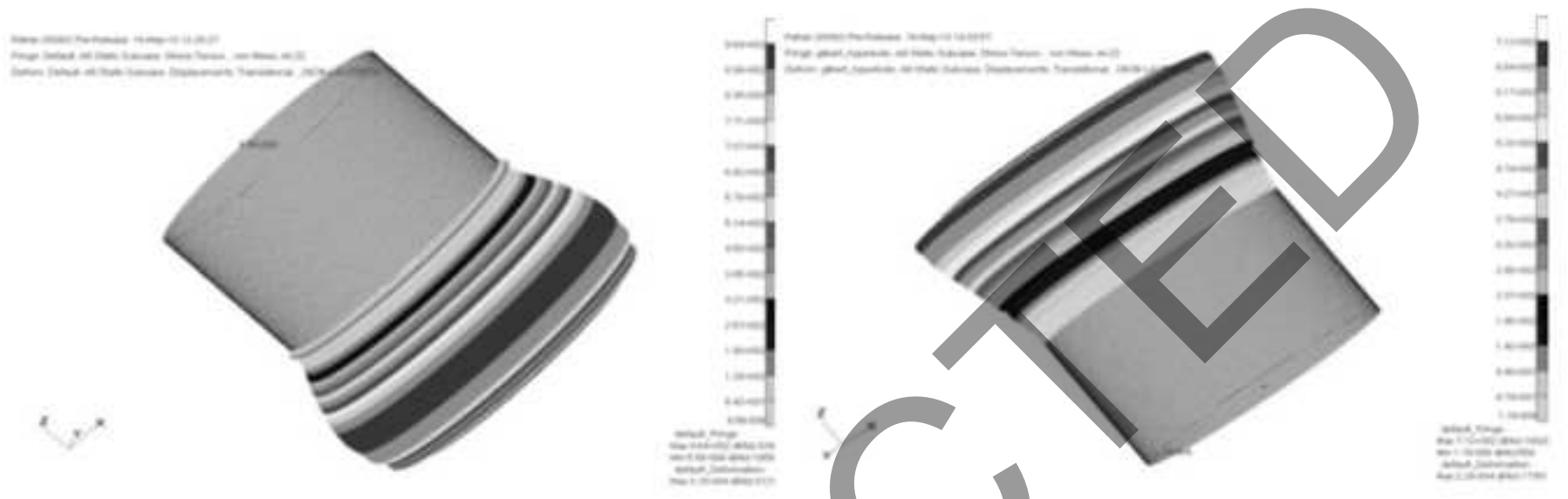


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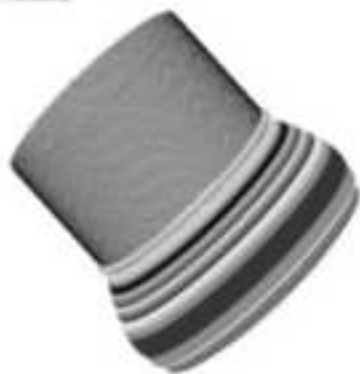
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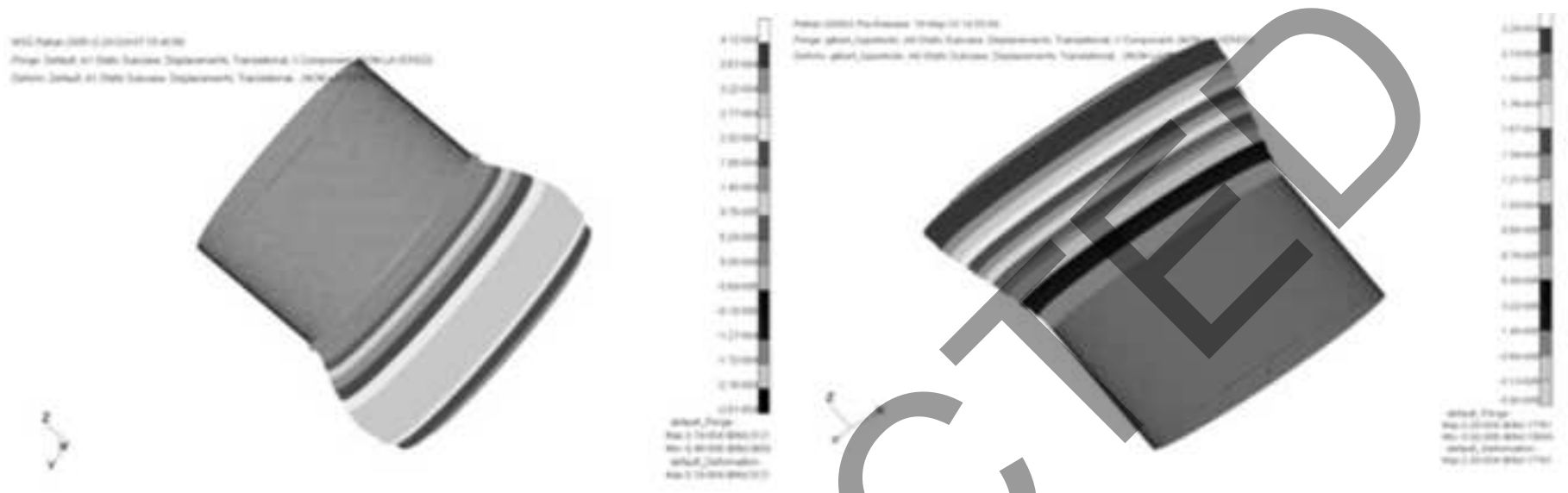


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FIGURE 1 The optical train and mirror cross section of a typical X-ray space telescope.

FIGURE 2 A thin shell mirror mounted in a support ring. Photo by Carl Benson, NASA/MSFC

FIGURE 3 Shell dimensions (left) and mirror cross section (right)

FIGURE 4 Parabolic shell profiles along four meridians

FIGURE 5 Hyperbolic shell profiles along four meridians

FIGURE 6 Finite Element Model

FIGURE 7 Residual von Mises stress plotted along a meridian for the parabolic (left) and hyperbolic (right) sections

FIGURE 8 Residual radial deformation plotted along a meridian for the parabolic (left) and hyperbolic (right) sections

FIGURE 9 Residual von Mises stress in the parabolic (left) and hyperbolic (right) sections

FIGURE 10 Residual Max Principal Stress in the parabolic (left) and hyperbolic (right) sections

FIGURE 11 Residual radial deformation in the parabolic (left) and hyperbolic (right) sections

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