

# *Slope Stability Analysis by the Limit Equilibrium Method*

## Companion Title

*LEAME Software and User's Manual: Analyzing Slope Stability by the Limit Equilibrium Method*, by Yang H. Huang, Sc.D., P.E. (ASCE Press, 2014). Offers a PC-based software program for performing two- and three-dimensional slope stability analyses, accompanied by supporting documentation and worked examples. (ISBN 978-0-7844-7799-1; online at <http://dx.doi.org/10.1061/9780784477991>)

## Other Titles of Interest

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# Preface

During the past 40 years, I have been engaged in a study on the stability analysis of earth slopes. The study was initiated in 1973 when I received a research grant from the Institute for Mining and Minerals Research, University of Kentucky. When the research project was completed in 1977, the U.S. Congress passed the Surface Mining Control and Reclamation Act, which requires stability analysis for refuse dams, hollow fills, and spoil banks created by surface mining, thus putting the research into practical use. The results of the research were published in several journals and reports and also were presented in a number of short courses. Both the simplified and the computerized methods of stability analysis, as developed from this research, have been widely used by practicing engineers throughout Kentucky for the application of mining permits. The large number of out-of-state participants in the short courses indicates that the methods developed have widespread applications.

In 1983, my book *Stability Analysis of Earth Slopes* was published by the Van Nostrand Reinhold Company. The book was well received by the engineering profession and was reprinted quite a few times. It was recommended by the Professional Civil Engineering Book Club as a feature selection and was translated into Chinese and Russian by foreign publishers. Two computer programs, one called SWASE (Sliding Wedge Analysis of Sidehill Embankments) for analyzing plane or noncircular failure surfaces, and the other called REAME (Rotational Equilibrium Analysis of Multilayered Embankments) for cylindrical failure surfaces, written in both Fortran and Basic languages, were listed in the book. In 1994, the SWASE program was incorporated into the REAME program, and a separate program for three-dimensional analysis, named REAME3D, was developed. In 1996, the first Windows version of REAME for both two- and three-dimensional analyses was released and used widely by the mining industries. The program has been continuously improved, and a new version has been released every four years, culminating in the latest REAME2012. Because the name REAME is a misnomer and the computer software associated with this book is very similar to REAME2012 with only some minor changes, the name REAME has been changed to LEAME (Limit Equilibrium Analysis of Multilayered Earthworks) in this book to reflect the capabilities of the software better.

Further evolutions of the book and software have prompted their separation into two separate products. *Slope Stability Analysis by the Limit Equilibrium Method: Fundamentals and Methods* presents the basic principles at work in the analysis of slope stability and common methods for analyzing slope stability in two and

three dimensions. A companion product, *LEAME Software and User's Manual: Analyzing Slope Stability by the Limit Equilibrium Method* provides both the software program and the supporting documentation for its use. The software can be obtained at <http://dx.doi.org/10.1061/9780784477991>.

Although some of the materials presented in this book, such as the five chapters in Part 1 and Chapter 7 in Part 2, are essentially the same as the 1983 book, this revised and updated volume is dramatically different in the following aspects:

1. Many new sections have been added, such as the back-calculation of shear strength, undrained shear strength varying linearly with depth, granular materials with curved strength envelope, unsteady-state seepage, and external and internal forces.
2. Some new stability charts have been added and some others have been deleted, because they are too cumbersome for hand calculations. With the availability of the LEAME software, no one likes to resort to stability charts for preliminary analysis unless they are very simple to use.
3. Only the limit equilibrium method is covered here, and the section on finite element method is eliminated. Also, only the methods incorporated in LEAME are presented in detail, while the sections on Janbu's method and Morgenstern's and Price's method are eliminated. The section on the probabilistic method has been expanded greatly, and a new chapter on reliability is presented.
4. The three-dimensional analysis, which was not even mentioned in the previous book, is presented here in a full chapter. It covers the theoretical aspect by showing how the failure mass is divided into columns and deriving the equations used for LEAME.
5. Spreadsheets have been added to solve many of the examples, and the results are compared with the LEAME computer program. It is amazing that many problems involving iterations or using trial-and-error can be solved easily by spreadsheets. Although spreadsheets can be used to check the correctness of a computer program, they cannot serve as a substitute, because they involve only a single failure surface; to determine the minimum factor of safety, hundreds of failure surfaces need to be analyzed.
6. Homework problems and more examples have been added so the book can serve as a college text for senior and graduate courses in soil mechanics and geotechnical engineering.

This volume is divided into two parts. Part 1 presents the fundamentals of slope stability and consists of five chapters. Chapter 1 describes slope movements and discusses some of the more well-known methods for stability analysis. Chapter 2 explains the mechanics of slope failures and defines the factor of safety for both cylindrical and plane failures. Chapter 3 discusses both the laboratory and the field methods for determining the shear strength of soils used for

stability analysis. Chapter 4 illustrates some methods for estimating the location of the phreatic surface and determining the pore pressure ratio. Chapter 5 outlines remedial measures for correcting slides.

Part 2 presents methods of stability analysis and also consists of five chapters. Chapter 6 derives some simple formulas for determining the factor of safety for plane failures. Chapter 7 presents a number of stability charts for determining the factor of safety for cylindrical failures, as well as the well-known friction circle and logarithmic spiral methods. Chapter 8 discusses methods of slices for two-dimensional analysis and derives the equations used in developing LEAME. Chapter 9 discusses the three-dimensional analysis with both ellipsoidal and planar ends and derives the equations used in LEAME. Chapter 10 discusses various methods to determine the reliability of slope design, including the use of Taylor's expansion for closed-form solutions and the mean-value first order second moment (MFOSM) method for computer solutions.

The Appendix provides an introduction to the LEAME software to encourage readers to obtain the software. The LEAME computer software is completely different from the REAME program listed in the 1983 book. It is an excellent and well-tested software program to determine the factors of safety for both two- and three-dimensional slopes and contains many new features not available elsewhere. It can be downloaded and used right away to solve practical problems in slope stability.

Finally, I want to thank ASCE Press for giving me the opportunity to publish these books. It is my sincere hope that the books, especially the LEAME computer software, will be helpful to civil engineers in their engineering practice and to college professors in teaching courses in slope stability.

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