Appendix 1

NOTATION

A	event label, as in "event A consists of all outcomes ω such that $X(\omega) \le x$"
a	a real number constant, as the degree-day snowmelt constant in Equations (5-2) and the probability limit in Equations (8-21)
a_g	*Climate Outlook* probability setting for average air temperature for period g in the lower third of its 1961–90 range; see Equation (4-2a)
B	event label, as in "event A is contained in event B"
BIAS_{XZ}	bias, the expected value of the difference between the random variables X and Z, $(X - Z)$, used to measure the difference between the centers of their distributions; see Equation (2-17c)
$\widehat{\text{BIAS}}_{XZ}$	sample bias, the sample average of the difference between the random variable values x and z, $(x - z)$, used as an estimate of BIAS_{XZ}; see Equation (2-20c)
b_g	*Climate Outlook* probability setting for average air temperature for period g in the upper third of its 1961–90 range; see Equation (4-2b)
c_g	*Climate Outlook* probability setting for average precipitation for period g in the lower third of its 1961–90 range; see Equation (4-2d)
D_i	snow water equivalent at the end of day i
d	the derivative operator; see Equation (2-13a) for an example
d_g	*Climate Outlook* probability setting for average precipitation for period g in the upper third of its 1961–90 range; see Equation (4-2e)
$E[\cdot]$	the "expected value" operator, defined as the integral of the operand over the probability distribution function; see Equations (2-15)
e_k	selected weights sum limit in kth equation in Equations (6-7), (6-10), and (6-11), corresponding to a climate outlook probability setting, as, for example, in Equations (6-8)
$F(x)$	cumulative distribution function, the probability that $X \le x$, as in Equation (2-19)
$f_X(x)$	the probability density function, the derivative of the cumulative distribution function, as in Equation (2-13a)
$f(x)$	penalty function, for use in optimization objective functions of Equations (8-22) and (8-23), for negative or zero arguments; see Equations (8-24)
f_i	forecast average lake level for month i; see Equations (11-1)–(11-11)

$g(x)$	penalty function, for use in optimization objective functions of Equations (8-22) and (8-23), for negative arguments; see Equations (8-24)
L	objective function (the Lagrangian) for an unconstrained optimization reformulated from the objective function for a constrained optimization by incorporating the constraints; see Equations (7-6) and (8-16)
ℓ_i	actual average lake level for month i; see Equations (11-1)–(11-11)
M_g	average basin moisture over period g
MSE_{XZ}	mean square error, the expected value of the square of the difference between the random variables X and Z, $(X - Z)^2$, used to characterize the difference between their values; see Equation (2-17d)
$\widehat{MSE}_{XZ}$	sample mean square error, the sample average of the square of the difference between the random variable values x and z, $(x - z)^2$, used as an estimate of MSE_{XZ}; see Equation (2-20d)
m	an integer constant, as the order of a value in a random sample in Equation (2-24) or (5-13), or the number of equalities to be satisfied in outlooks derivative to meteorology outlooks in Equations (6-10), (6-11), and (8-10)
m_g	a value of average basin moisture for period g; see Equation (3-13) or (3-15)
$\hat{m}_{g,\gamma}$	reference average basin moisture γ-probability quantile for period g; see Equations (3-12) and (3-14)
N	number of duplicated scenarios in the hypothetical very large structured set used for estimation in the operational hydrology outlook
N_L	number of duplicated scenarios, in the hypothetical very large structured set used for estimation in the operational hydrology outlook, which have $Q_{\text{Sep}} \leq \hat{\theta}_{\text{Sep},0.333}$ in Equation (5-21a)
n	number of scenarios available for use in generating the operational hydrology outlook; see Equation (5-8)
$n_{(\text{A})}$	number of scenarios, available for use in generating the operational hydrology outlook, in which event A occurs (A is true) in Equations (5-27) and (5-28)
n_L	number of scenarios, available for use in generating the operational hydrology outlook, which have $Q_{\text{Sep}} \leq \hat{\theta}_{\text{Sep},0.333}$ in Equation (5-22a)
$n_{X \leq x}$	number of observations that are less than or equal to x, used in estimating the probability of nonexceedance with the sample relative frequency; see Equation (2-22)
$P[\cdot]$	probability of the event in brackets, representing its likelihood
$P[X \leq x]$	cumulative distribution function, the probability that $X \leq x$, as in Equation (2-9)
$\hat{P}[\cdot]$	relative frequency in a set, of the event in brackets, used as a probability estimate

p | an integer constant, as the number of strictly-less-than probability constraint inequalities selected from probabilistic meteorology outlooks for use in making an operational hydrology outlook; see Equations (8-9) and (8-10)

Q_g | total precipitation over period g

$Q_{h,g}$ | total precipitation in application area h over period g, as $Q_{\text{Sup, SON98}}$ represents the Lake Superior application area and the September-October-November 1998 period

q | an integer constant, as the number of less-than-or-equal-to probability constraint inequalities selected from probabilistic meteorology outlooks for use in making an operational hydrology outlook; see Equations (8-9) and (8-10)

$(q_g)_i$ | value of Q_g in scenario i

R_g | total basin runoff over period g

r | number of interval limits defining the $r + 1$ intervals of the real line for a variable's values; used additionally to denote the hydrology model in Equation (6-24)

$(r_g)_i$ | value of R_g in scenario i

S | vector of modeled hydrological output variables for the Bayesian Forecasting System

s | vector of values of modeled hydrological outputs for the Bayesian Forecasting System; see Equation (6-24)

s_X^2 | sample variance, the sample average of the square of the difference between random variable values x and the sample mean $\bar{x}$, $(x - \bar{x})^2$, used as an estimate of σ_X^2; see Equation (2-20b)

s_Z^2 | sample variance, the sample average of the square of the difference between random variable values z and the sample mean $\bar{z}$, $(z - \bar{z})^2$, used as an estimate of σ_Z^2; see Equation (2-20b)

T_g | average air temperature over period g

$T_{h,g}$ | average air temperature in application area h over period g, as $T_{\text{Sup, SON98}}$ represents the "Superior" application area and the September-October-November 1998 period

$(t_g)_i$ | value of T_g in scenario i

u | an integer constant, as the number of probability constraint inequalities selected from probabilistic meteorology outlooks for use in making an operational hydrology outlook; see Equation (10-15c)

u | vector of deterministic inputs to a hydrology model for the Bayesian Forecasting System

V | vector of meteorology parameters of a probabilistic quantitative forecast for the Bayesian Forecasting System

v | vector of values of meteorology parameters of a probabilistic quantitative forecast used in the Bayesian Forecasting System

v | "artificial" variable to drive to zero in Equations (10-19)

W | vector of meteorology input variables for the Bayesian Forecasting System

W_g	average water surface temperature over period g
$\mathbf{w}$	vector of values of meteorology inputs for the Bayesian Forecasting System
$\left(w_g\right)_i$	value of W_g in scenario i
w_i	weight applied to ith scenario in the original set of possible future scenarios for calculation of estimates in a derivative outlook, or added "slack variable" to convert an inequality to an equation
X	a meteorology or hydrology variable
x_i	value for variable X in ith scenario in the set of possible future scenarios
x_k^N	value for variable X in kth duplicated scenario in the hypothetical very large structured set of N scenarios
x_i^n	value for variable X in ith scenario in the original set of n possible future scenarios
$\overline{x}$	sample mean, the sample average of random variable values x, used as an estimate of μ_X; see Equation (2-20a)
y_j	jth ordered value, from smallest to largest, for variable X in the set of possible future scenarios; see Equation (2-24)
y_j^N	jth ordered value, from smallest to largest, for variable X in the hypothetical very large structured set of N scenarios
y_j^n	jth ordered value, from smallest to largest, for variable X in the set of n possible future scenarios
Z	a meteorology or hydrology variable
z_j	value for variable Z in jth scenario in the set of possible future scenarios; used additionally in Equations (8-1) through (8-7) as the jth interval limit (j = 1, … , r) defining the r + 1 intervals of the real line for a variable's values
z_i^n	value for variable Z in ith scenario in the original set of n possible future scenarios
$\overline{z}$	sample mean, the sample average of random variable values z, used as an estimate of μ_Z; see Equation (2-20a)
α_i	an integer coefficient equal to unity (1) if and only if the defining event is true in scenario i and zero (0) otherwise; see, for example the transforming of Equations (6-2) and (6-3) into Equations (6-4) and (6-5)
$\alpha_{k,i}$	an integer coefficient equal to unity (1) if and only if defining event k is true in scenario i and zero (0) otherwise; see, for example the transforming of Equations (6-6) and (6-3) into Equations (6-7) and (6-9); becomes then the coefficient in the kth equation on the ith weight (for the ith scenario) in Equation (6-7)
∂	the partial derivative operator; see Equations (7-7) and (8-17) for examples
γ	a real number (probability) between zero (0) and unity (1), inclusively
η	probability density of **W**, or of **W** given **V**, used in the Bayesian Forecasting System

ϑ_i duplication count for ith scenario in the original set of possible future scenarios for the hypothetical very large structured set

$\hat{\theta}_{g,\gamma}$ reference total precipitation γ-probability quantile for period g; see Equation (3-9)

$\hat{\theta}_{h,g,\gamma}$ reference total precipitation γ-probability quantile in application area h over period g, as $\hat{\theta}_{\text{Sup, SON98, 0.667}}$ represents the 0.667-probability quantile for the Lake Superior application area and the September-October-November 1998 period

λ_k LaGrange multiplier, representing the penalty associated with violation of the kth constraint equation in an optimization

μ_X mean, the expected value of random variable X used to define the central location of its distribution; see Equation (2-17a)

μ_Z mean, the expected value of random variable Z used to define the central location of its distribution; see Equation (2-17a)

ξ_γ reference γ-probability quantile for variable X

$\prod$ the product operator (product of expression following the operator indexed as indicated in the operator subscripts and superscripts); see Equation (2-19) for example

π probability density of **S** given **u** used in the Bayesian Forecasting System

ρ_{XZ} correlation, the expected value of the normalized product of the difference between random variable X and its mean $(X - \mu_X)$ and the difference between random variable Z and its mean $(Z - \mu_Z)$, used to characterize the linear dependence of one on another; see Equation (2-17e)

$\hat{\rho}_{XZ}$ sample correlation, the sample average of the normalized product of the difference between random variable values x and the sample mean $(x - \bar{x})$ and the difference between random variable values z and the sample mean $(z - \bar{z})$, used as an estimate of ρ_{XZ} ; see Equation (2-20e)

$\sum$ the summation operator (sum of expression following the operator indexed as indicated in the operator subscripts and superscripts); see Equation (2-14) for an example

σ_X^2 variance, the expected value of the square of the difference between random variable X and the mean $(X - \mu_X)^2$ used to define the spread of the distribution about its central location; see Equation (2-17b)

σ_Z^2 variance, the expected value of the square of the difference between random variable Z and the mean $(Z - \mu_Z)^2$ used to define the spread of the distribution about its central location; see Equation (2-17b)

$\hat{\tau}_{g,\gamma}$ reference average air temperature γ-probability quantile for period g; see Equation (3-5) for an example with $\gamma = 0.50$

$\hat{\tau}_{h,g,\gamma}$ reference average air temperature γ-probability quantile in application area h over period g, as $\hat{\tau}_{\text{Sup, SON98}, 0.667}$ represents the 0.667-probability quantile for the Lake Superior application area and the September-October-November 1998 period

ϕ joint probability density of **W** and **V** for the Bayesian Forecasting System

ϕ_j probability limit for a most-probable interval outlook for interval j

Ω population or target space, the set of all possible outcomes in an experiment or the set of all possibilities in a population

ω possibility or outcome, an element from Ω

$\forall$ "for all"; see Equation (2-2) for an example

$\int$ the integral operator (integral of the expression following the operator over the limits indicated in the operator subscripts and superscripts); see Equation (2-13b) for an example

$\varnothing$ the null set or empty set; see Equation (2-1d)

$\cap$ the intersection operator, denoting the intersection of sets; see Equation (2-1b) for an example

$\cup$ the union operator, denoting the union of sets; see Equation (2-1a) for an example

$\uplus$ the exclusive union operator, denoting the exclusive union of sets; see Equation (2-4a) for an example

^{c} the complement operator, denoting the complement of a set; see Equation (2-1c) for an example

$\subset$ containment of one set in another; see Equation (2-2) for an example

$\in$ membership in a set; see Equations (2-1) for an example

$\Leftrightarrow$ "if and only if"; see Equation (2-2) for an example

$\Rightarrow$ "implies"; see Equation (2-2) for an example