Friction Modeling of Flood Flow Simulations

Vasilis Bellos[1]; Ioannis Nalbantis[2]; and George Tsakiris[3]

Abstract: A new, three-parameter friction model at the boundaries of free surface flow is proposed. The model is valid for all flow and roughness characteristics and it is proposed to replace the widely used Manning equation in the flood simulation models. In the gauged flow domains, the parameters of the model can be calibrated directly using the appropriate field data. For the ungauged flow domains, the parameters are estimated following a four-step procedure: (1) selection of roughness zones; (2) generation of synthetic data from water depth—flow velocity—roughness height combinations; (3) calculation of shear stresses using a physically based equation; and (4) estimation of the model parameters through regression toward the results of this equation. The proposed friction model is used in the case study of the Tous dam break in Spain in 1982, as it was simulated by the two-dimensional hydrodynamic model FLOW-R2D. The results of this application show that the proposed friction model performs slightly better than the commonly used Manning equation. **DOI: 10.1061/(ASCE)HY.1943-7900.0001540.** *This work is made available under the terms of the Creative Commons Attribution 4.0 International license, http://creativecommons.org/licenses/by/4.0/.*

Introduction

The shear stress created in a fluid body during its motion along a solid boundary, known also as friction, is a very important factor in the simulation of flood events. It may be the primary source of uncertainty in the results of flood simulation. This is even more profound in flood flows occurring in mild terrains, in which other parameters such as topography play a minor role.

So far, various friction models have been proposed and used in practical applications, which have been derived from the theory of fluid mechanics (e.g., Bates et al. 2005; Cheng 2008; López et al. 2009; Nepf 2012; Özgen et al. 2015). However, so far, this theoretical knowledge has not yet been exploited for improving hydrodynamic modeling. The majority of the numerical flood simulation models [one-dimensional (1D) or two-dimensional (2D)] adopt semi-empirical equations derived in the 19th century, such as the Manning equation. As known, the Manning equation needs only one friction factor, which can be estimated easily from the literature, practically for any terrain.

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Obviously, the Manning equation is understood as a gross procedure, since its friction coefficient is assumed to be constant, regardless of both water depth and flow velocity. This fact has been questioned recently (Ferguson 2010). Besides, many modelers attempt to calibrate friction coefficients based on historical data flood events or from field data (Papanicolaou et al. 2011). Yet, the optimized Manning friction coefficient values are usually greater than those encountered in the literature, because they incorporate turbulence energy losses (Jarrett 1985; Christelis et al. 2016). This is because the flow during flood events is so *violent* in comparison with the physical models that the energy losses from turbulence are reflected in the increased friction coefficients (Morvan et al. 2008).

This fact led us to consider friction coefficients as grey-box parameters, which means that the values found in the scientific literature are not adopted as globally valid, but they cannot be considered as black-box parameters either (Kroll 2000).

By adopting this nature of friction parameters, a new, three-parameter friction model is presented, in which the shear stresses are assumed proportional to a power of flow velocity and reciprocal to a power of water depth.

For convenience in the analysis, the flow domains in two categories are distinguished: (1) the gauged flow domains in which all necessary data exist, so that the calibration process can be performed; and (2) the ungauged flow domains, in which data are not available. The term *flow domain* means the natural free-surface flow in several cases, such as flow in fluvial scale, floodplains, urban environments, etc.

For the first category, the process for defining the friction parameters is rather simple and involves two steps: (1) development of an objective function for the comparison between observed and simulated data (on the basis of the available data, such as observed maximum water depth, flow rate measurements, etc.); and (2) calibration of friction parameters through optimization of this objective function.

For the second category, we transfer the scientific knowledge on bottom shear stress from the microscale of fluid mechanics, to the scales used in the hydrodynamic modeling.

In this context, a new explicit equation for the friction factor f of the Darcy-Weisbach equation is derived to represent the bottom shear stresses in a more detailed fashion. Further, the ungauged flow

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domains are classified with respect to the roughness, and the three parameters for each roughness class are estimated.

Three-Parameter Friction Model

Using the principle of momentum conservation and assuming a 2D approach (in which the hydraulic radius is equal to water depth), Eq. (1) can be written for the shear stresses at the bottom τ_b as follows:

$$\frac{\tau_b}{\rho} = ghS_f \tag{1}$$

where ρ = the density of the fluid; h = the water depth; and S_f = the energy slope.

By replacing S_f from the Manning equation, Eq. (1) becomes:

$$\frac{\tau_b}{\rho} = \frac{gn^2V^2}{h^{1/3}} \tag{2}$$

where n = the Manning coefficient; and V = the flow velocity.

To transform the Eq. (2) for 1D analysis, the water depth should be replaced by the hydraulic radius R. Eq. (2) represents the most common approach adopted in the majority of the hydrodynamic software packages.

As mentioned, a new, three-parameter friction model is proposed in which shear stresses are proportional to a power of flow velocity and reciprocal to a power of water depth, as follows:

$$\frac{\tau_b}{\rho} = \frac{V^A}{Bh^C} \tag{3}$$

where A, B, C are the three parameters of the new friction model.

By analyzing Eq. (3) in the following way:

$$\log_{10}\left(\frac{\tau_b}{\rho}\right) = \alpha\log_{10}(h^cV^d) + b = \log_{10}(10^b h^{\alpha c}V^{\alpha d}) \Rightarrow \frac{\tau_b}{\rho} = 10^b h^{\alpha c}V^{\alpha d} \tag{4}$$

Parameters A, B, C can be estimated as follows:

$$A = ad, \qquad B = \frac{1}{10^b}, \qquad C = -ac \tag{5}$$

For $A = 2$, $C = 1/3$, and $B = 1/gn^2$, Eq. (3) becomes identical to Eq. (2), which is based on the Manning equation. Therefore, it can be concluded that Eq. (2) is a special case of the proposed general friction model, which incorporates three parameters instead of one.

Analysis of a Gauged Flow Domain Case Study

In the gauged flow domains, the friction parameters are implicitly calibrated (since shear stresses cannot be directly measured), by adopting physical constraints based on prior scientific knowledge. For illustrative purposes, a case study is presented as an application example for the friction model implementation. The example refers to the flood wave caused by the dam break of the Tous dam in Spain in 1982, as it was reconstructed by Alcrudo and Mulet (2007).

This case study is a well-known benchmark test for flood simulation models and has been used in previous works, mainly related to the application and testing of the 2D hydrodynamic model FLOW-R2D (Tsakiris and Bellos 2014; Bellos and Tsakiris 2015b, a, 2016). This model solves the 2D shallow water equations (2D–SWE) through the finite difference method using a modified version of the McCormack numerical scheme.

In the past, an effort was made using surrogate modeling for the calibration of the Manning n value for the entire computational domain and the effective slope (S_{eff}) used in the upstream boundary condition (Christelis et al. 2016).

In this effort, the objective function was the sum of the squared errors (SE) between the observed and simulated water depths at the 21 water depth gauges in Sumacárcel, located a close distance downstream of the dam. The results derived from this optimization procedure for the SE value was 37.889 m^2 and the calibrated quantities were $n = 0.194\ \mathrm{s/m}^{1/3}$ and $S_{\mathrm{eff}} = 0.019$.

In the present study, the same surrogate model is implemented, but instead of the Manning equation, the proposed friction model is used. In this case, the same optimization procedure leads to a slightly lower value of SE (37.151 m^2), whereas the calibrated parameters are $A = 2.596$; $B = 10.0$; $C = 0.1$; and $S_{eff} = 0.018$.

Admittedly, the decrease of SE is relatively small (approximately 2%), but probably because of the limited computational budget (100 simulations), and the fact that only two parameters are calibrated for the Manning equation, whereas four parameters are determined for the proposed friction model. Needless to say, the proposed method needs to be tested further, in several applications.

Regarding this application, Fig. 1 presents the water depths and flow velocities derived by the FLOW-R2D model combined with the proposed friction model. In Fig. 2, the observed maximum water depths of the proposed friction model are compared with those derived by the same hydrodynamic model using the Manning equation.

Analysis for Ungauged Flow Domains

General

The case of ungauged flow domains is the most common one, since there are very few case studies with observed data and even fewer studies with continuous monitoring. Hence, the only possible procedure for acquiring a rational friction model is to take advantage of the theory in parallel with the collection of data of the flow domain characteristics (e.g., granulometry, vegetation, buildings, etc.) using advanced techniques such as photointerpretation.

An application example of the proposed friction model in ungauged flow domains follows. It involves the classification of several flow domains and estimation of the three parameters A, B, C, through a new equation for the calculation of the Darcy-Weisbach friction factor f. The equation is physically based, explicit, and unconditionally valid for both flow regimes (laminar and turbulent). The qualifier *physically based* means that this developed equation depends on both the flow characteristics (water depth, flow velocity, and kinematic viscosity) and the flow domain characteristics that can be measured (e.g., roughness height). Therefore, the new equation is quite suitable for modeling, especially in cases in which the roughness height can be estimated with reasonable accuracy.

For the ungauged flow domains, the theory was exploited from the microscale of fluid mechanics to reach scales used in the hydrodynamic models, using two steps.

In the first step, a new explicit equation for the Darcy-Weisbach friction factor f equation is derived to represent the bottom shear stresses. This equation is valid for all flow conditions (laminar and turbulent) and appropriate for free-surface flow. The flow characteristics and the roughness height are the input data for the proposed equation, which is obtained combining elements from the boundary

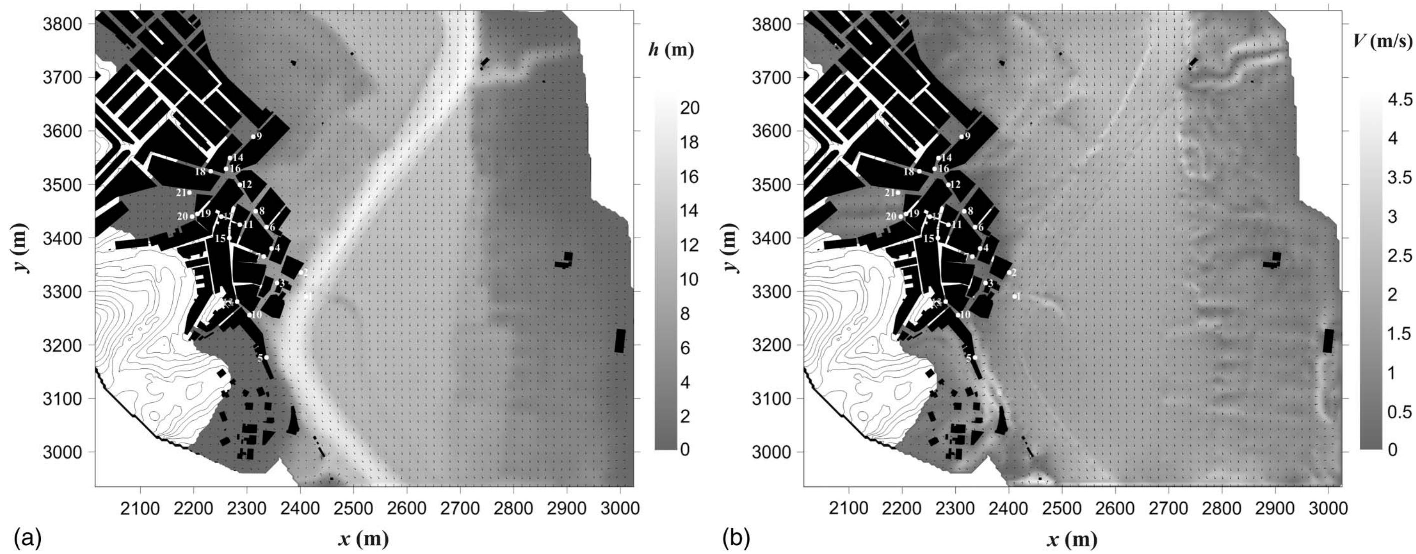

Fig. 1. (a) Water depths; and (b) flow velocities simulated using the proposed friction model.

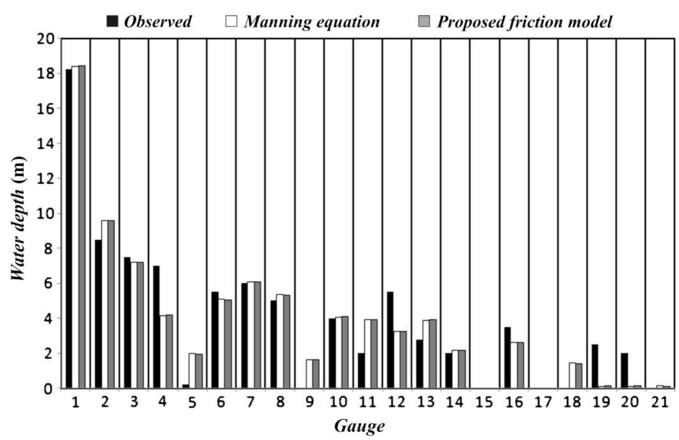

Fig. 2. Comparison between the maximum water depths recorded, the maximum water depths simulated by the Manning equation, and the maximum water depths obtained using the proposed friction model, at the 21 water level gauges in the floodplain downstream the Tous dam.

layer theory for a solid boundary and the well-known experimental data published by Nikuradse (1933).

The second step deals with the classification of the ungauged flow domains with respect to roughness and the estimation of model parameters. This step incorporates the following actions: (1) roughness height range for several domains is estimated based on the related literature; (2) random combinations of water depth, flow velocity, and roughness height are generated using the Monte Carlo simulation based on the uniform probability distribution with preassigned ranges of involved variables; (3) based on the aforementioned synthetic data, shear stresses are determined using the new, analytical relationship derived in the first step; and (4) the three parameters are determined based on the best fit of the synthetic data toward the results derived using the analytical relationship.

New Friction Equation for Free Surface Flow

The new explicit equation is presented here for estimating Darcy-Weisbach friction factor f in free-surface flows, valid under all flow regimes (from laminar to turbulent flow), and which depends on flow characteristics (water depth and flow velocity) and roughness height. The analytical process in which this equation is derived is described in the Appendix and is based on Cheng's idea (Cheng 2008). Based on this process, friction factor f can be calculated as follows:

$$f = \left(\frac{24}{Re_h}\right)^{\alpha}\left[\frac{0.86e^{W(1.35\mathsf{R}_h)}}{\mathsf{R}_h}\right]^{2(1-\alpha)b}\left\{\frac{1.34}{\left[\ln\left(12.21\frac{h}{k_s}\right)\right]^2}\right\}^{(1-\alpha)(1-b)} \tag{6}$$

where

$$\alpha = \frac{1}{1+\left(\frac{\mathsf{R}_h}{678}\right)^{8.4}} \tag{7}$$

$$b = \frac{1}{1+\left(\frac{\mathsf{R}_h}{150\frac{h}{k_s}}\right)^{1.8}} \tag{8}$$

$$W(1.35\mathsf{R}_h) = \ln(1.35\mathsf{R}_h) - \ln[\ln(1.35\mathsf{R}_h)] + \frac{\ln[\ln(1.35\mathsf{R}_h)]}{\ln(1.35\mathsf{R}_h)} + \frac{\ln[\ln(1.35R_h)]^2 - 2\ln[\ln(1.35R_h)]}{2[\ln(1.35R_h)]^2} \tag{9}$$

and R_h = the Reynolds number assuming that the characteristic length of the flow is the water depth h; while k_s = the roughness height.

Having obtained f from Eq. (6), the bottom shear stresses can be calculated easily using the Darcy-Weisbach equation, combined with Eq. (1) as follows:

$$\frac{\tau_b}{\rho} = \frac{f}{8}V^2 \tag{10}$$

Classification of Flow Domains

As mentioned, the classification of flow domains based on the level of roughness consists of the following steps:

Table 1. Parameters A, B, and C of the proposed friction model and Manning n values for various flow domains

Zone	Roughness height (mm)	Reference	Parameters of the proposed friction model			NSE	Manning coefficient n	NSE
			A	B	C			
Silt	0.0039–0.0625	Krumbein and Aberdeen (1937)	3.355	58,328.9	0.169	0.54	0.0061	0.45
Concrete	0.3–3	Chanson (2004)	2.001	475.9	0.220	0.97	0.0156	0.91
Untreated shot concrete	3–10	Chanson (2004)	1.995	318.8	0.264	0.98	0.0184	0.96
Rubble masonry	5–10	Chanson (2004)	1.994	304.3	0.269	0.99	0.0187	0.97
Asphalt	1–1.5	ELOT (2009)	2.007	496.6	0.211	1.00	0.0153	0.92
Fine sand	0.0625–0.5	Krumbein and Aberdeen (1937)	2.155	1,004.9	0.184	0.95	0.0128	0.84
Coarse sand	0.5–2	Krumbein and Aberdeen (1937)	2.016	507.4	0.214	0.99	0.0152	0.91
Sand	0.0625–2	Krumbein and Aberdeen (1937)	2.041	579.7	0.206	0.95	0.0147	0.87
Fine gravel	2–16	Krumbein and Aberdeen (1937)	2.000	292.7	0.286	0.95	0.0190	0.94
Medium coarse gravel	16–32	Krumbein and Aberdeen (1937)	2.007	209.5	0.347	0.97	0.0221	0.97
Very coarse gravel	32–64	Krumbein and Aberdeen (1937)	1.997	152.4	0.431	0.93	0.0251	0.88
Coarse gravel	16–64	Krumbein and Aberdeen (1937)	2.057	187.9	0.410	0.91	0.0242	0.88
Gravel	2–64	Krumbein and Aberdeen (1937)	1.980	180.2	0.377	0.86	0.0230	0.85
Cobble	64–256	Krumbein and Aberdeen (1937)	1.834	17,580.2	2.655	0.50	0.0376	0.09

1. Based on literature, various roughness zones in the form of intervals for roughness height are produced for each one of the examined materials; an example of this classification is presented in Table 1. It is noted that vegetation or buildings are not considered in the analysis.
2. A synthetic data set is generated for each class of roughness height using the Monte Carlo simulation: 100,000 combinations of water depth, flow velocity, and roughness height are randomly generated from a uniform probability distribution, using prespecified intervals for water depth and flow velocity. These are selected to cover the possible range of the two variables in real applications. Specifically, for water depth, the interval is 0.02–20 m while the corresponding range for flow velocity is 0.02–10 m/s. The kinematic viscosity is assumed to be constant and equal to 10^{-6} m^2/s. The range for roughness height is determined from the previous step in each zone.
3. Shear stresses are determined using Eq. (10), calculating the friction factor f with the Eq. (6).
4. The three parameters are determined based on the best fit between the results obtained from Eq. (10) [the friction factor f being calculated from the Eq. (6)] and the results of the proposed friction model [Eq. (3)]. The fit is achieved using the well-known generalized reduced gradient or GRG solving method (Lasdon et al. 1978), through the maximization of the Nash-Sutcliffe efficiency coefficient (*NSE*). The optimal values of the three parameters and the *NSE* metric, obtained for each roughness zone, are presented in Table 1. Apart from the three parameters, the Manning n value is calculated following the same process as previously mentioned (Table 1). In Fig. 3, the results from the fitting of the three parameters, for various zones, are presented.

The aforementioned classification can also be used for checking the plausibility of the calibrated friction parameters in cases of gauged flow domains. Since friction parameters are grey-box, the calibrated values can be tested as to whether they take realistic values or not.

Finally, the issue of uncertainty of friction model parameters is crucial when comparing models, since *interval estimates* of parameters normally lead to safer conclusions than *point* estimates do. However, in this work, our analysis is limited to *point* estimates because of the computational budget limitations, for both Manning and the proposed friction model. Needless to say, approaches that are less demanding in computing time can afford the assessment of uncertainty of model parameters (Nalbantis and Lymperopoylos 2012; Nalbantis et al. 2017).

Discussion

Why Is It Necessary for the New Equation to be Valid under All Flow Regimes?

In this section, an example is provided to show the advantage of an equation valid under all flow regimes, against the use of one of the several equations for calculating the Darcy-Weisbach friction factor. The majority of the proposed relationships cover a part of flow regimes, such as the well-known Colebrook-White equation (Colebrook 1939), which is known to be valid only for turbulent flow (i.e., with R_D greater than 4,300).

The following simple numerical example can reveal the problems created when explicit relationships developed for a specific flow regime are applied in other regimes. See, for example, the Swamee and Jain equation (Swamee and Jain 1976), in relation to flow with low Reynolds numbers:

$$f = \frac{0.25}{\left\{\log\left[\frac{5.74}{(4\mathsf{R}_h)^{0.9}} + \frac{k_s/4h}{3.7}\right]\right\}^2} \tag{11}$$

If $k_s = 0.05$ m, $v_s = 10^{-6}$ m^2/s, and $V = 0.02$ m/s, the friction factor plotted versus R_h for both approaches (the proposed equation and the Swamee and Jain equation) appears in Fig. 4. In cases of low Reynolds number and water depths smaller than the roughness height, the friction factor calculated by the Swamee and Jain equation takes unrealistic values.

Although low Reynolds numbers are not common in practice, and water depths are usually greater than the roughness height, during the numerical simulation of a flood wave, there might be cells or elements in which flow conditions are characterized by the aforementioned conditions. Examples of such situations can be encountered in cases in which water has been trapped in a *blind spot* of the topography and remains semi-still.

Is a Homogeneous Domain Really Homogeneous?

Another important issue is the test of homogeneity with respect to the friction characteristics of an area. Again, a simple example is used here to show that even for relatively homogeneous materials, the range of roughness height affects the computational results.

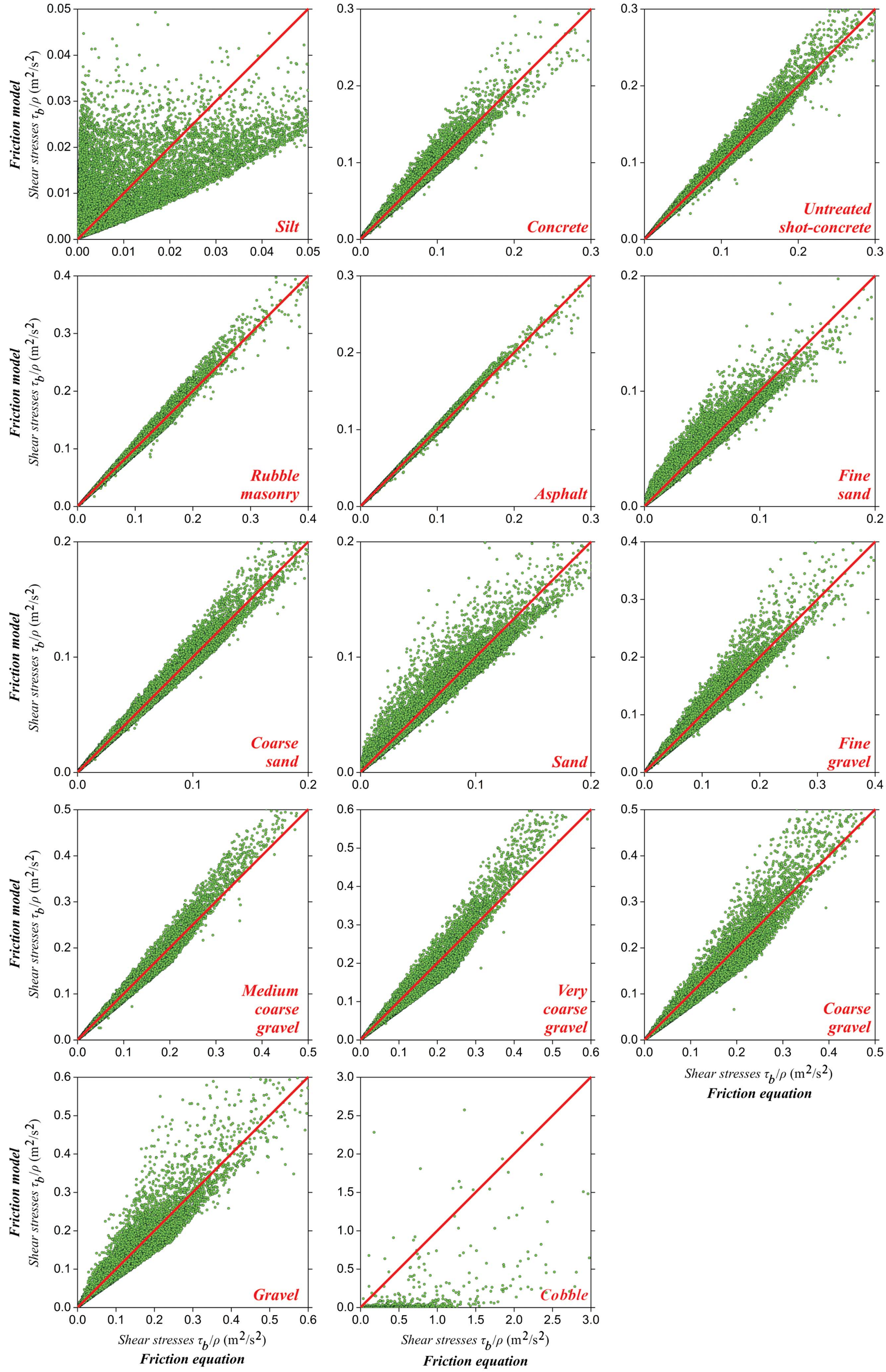

Fig. 3. Comparison of shear stresses calculated via the proposed friction model and the new friction equation for various flow domains.

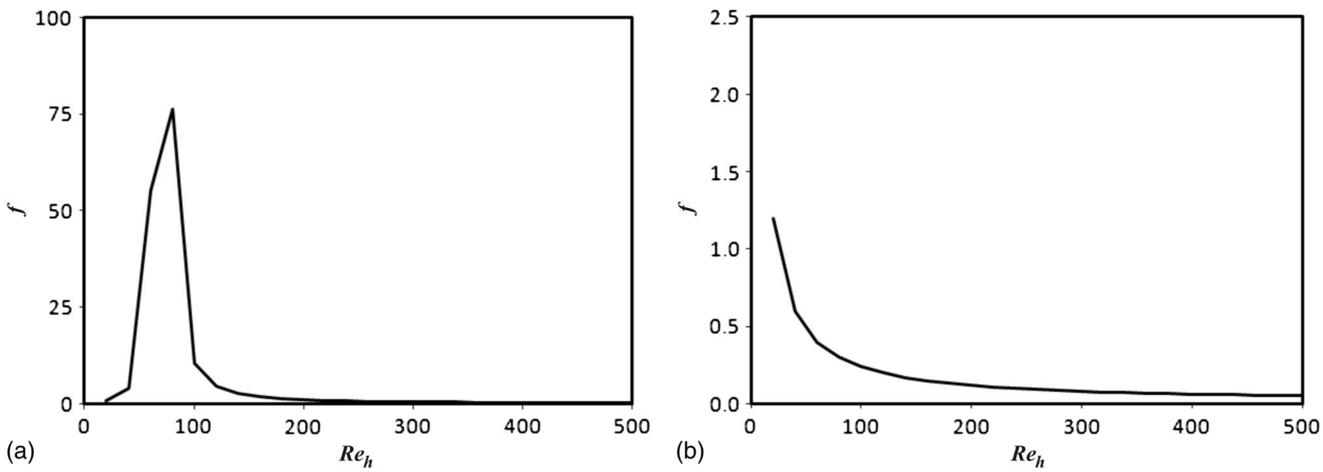

Fig. 4. Friction factor versus the Reynolds number in an example using: (a) the Swamee and Jain (1976); and (b) the new friction equation.

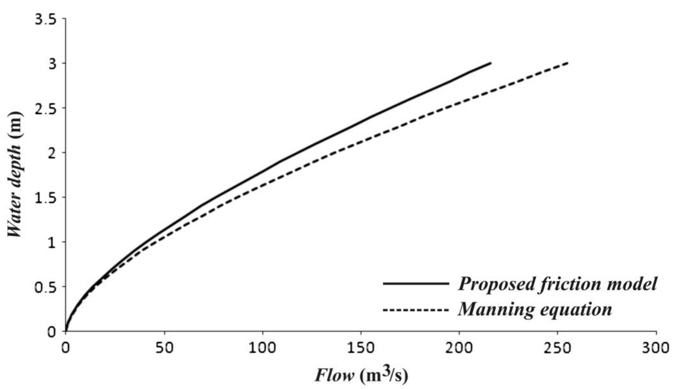

Fig. 5. Comparison of stage-discharge curves obtained using the proposed friction model and the Manning equation.

Coupling Eqs. (1) and (3), the steady flow rate Q in a channel can be calculated as follows:

$$Q = A_w (gBR^{C+1}S_0)^{\frac{1}{A}} \tag{12}$$

where A_w = the wetted area of the cross section; and S_0 = the constant slope of the channel.

In Fig. 5, flow rate vs. water depth relationships are compared (known as the stage-discharge curves) for the steady flow obtained through Eq. (12) and the Manning equation, for a rectangular channel 20 m wide with a constant slope 1‰. The channel is made of concrete; therefore, a typical value for the Manning roughness coefficient is used ($n = 0.013\ \mathrm{s/m^{1/3}}$; Chaudhry 2008). The values for the three parameters A, B, C are taken from Table 1. It can be observed that there are significant deviations between the two curves, especially for higher value flow rates. Therefore, it is concluded that the assumption of roughness homogeneity and the selection of a unique value for Manning coefficient in a flow domain needs careful consideration.

Concluding Remarks

The objective of this study was to develop a new friction model, suitable for incorporation into 2D numerical flood simulation models. In this context, the following concluding remarks can be drawn:

The friction parameters incorporated in flood simulation models can be considered as grey-box parameters, which can be calibrated if data are available. The proposed three-parameter friction model proved to perform better than the Manning equation using data from a real flood event. However, additional systematic efforts should be undertaken covering tests in a wide range of domain types before this conclusion is generally accepted.

For the ungauged flow domains, an attempt was made to transfer the knowledge on shear stress from fine scales to large scales used in practical applications. A new, physically based, explicit equation was derived, suitable for free surface flows, which expresses the Darcy-Weisbach friction factor as a function of the flow characteristics and the roughness height. This equation was then used for the classification of flow domains with respect to roughness.

Appendix

Basic Notions

Shear stress is known to be proportional to the square of the mean flow velocity. This means that for determining the friction effect on the flow, the velocity profile should be known. In fact, the full velocity vector in three dimensions [three-dimensional (3D) approach] is required. Alternatively, the mean flow velocity with respect to the vertical axis (in 2D approach), or the mean flow velocity of the entire cross section (in 1D approach), are required.

According to the fluid dynamics theory, a boundary layer is formed in the flow near the solid boundaries with its thickness directly dependent on the flow velocity.

The basic assumptions related to this boundary layer, which are appropriate for the derivation of the new friction equation, are the following: (1) the thickness of the boundary layer is stable and coincides with the entire flow profile; (2) for the laminar flow, the velocity profile is parabolic; and (3) for the turbulent flow, the flow velocity becomes zero at a distance from the boundary equal to z_0; a laminar viscous sublayer is formed beyond this distance with a thickness δ, while, within this sublayer, the velocity profile is linear. In the remaining part of the layer, the flow is turbulent and the velocity profile is logarithmic (law of the wall).

As known, the shear velocity, V_*, and Reynolds number R_L, are defined as follows:

$$V_* = \sqrt{\frac{\tau_b}{\rho}} \tag{13}$$

$$\mathsf{R}_L = \frac{\bar{V}L}{v_s} \tag{14}$$

where $\bar{V}$ = the mean flow velocity; L = the characteristic flow length, such as hydraulic radius, water depth, pipe diameter, etc.; and v_s = the kinematic viscosity of the fluid. The use of Eq. (13) implies that the momentum conservation equation for uniform, steady flow is always valid.

Since the objective of this study was to examine the bottom friction in a free-surface flow, modeled through a 2D approach, the mean velocity with respect to the vertical (z) axis has to be determined. For this purpose, data from Nikuradse's experiments are used, which were initially derived in circular pipes with pressure-driven flow.

The laminar flow velocity profile on the vertical axis z for a circular pipe (where $z = 0$ at the bottom of the pipe) and an infinite width channel, are given by the following equations for pressure-driven flow in pipes and free-surface flow, respectively:

Pressure-driven

$$V = \frac{gS_f}{v_s}\left(\frac{Dz - z^2}{4}\right) \tag{15}$$

Free-surface

$$V = \frac{gS_f}{v_s}\left(\frac{2hz - z^2}{2}\right) \tag{16}$$

where D = the pipe diameter; and h = the water depth.

By integrating the aforementioned equations with respect to the corresponding cross section, the mean velocity $\bar{V}$ for the pressure-driven and free-surface flows are expressed, respectively:

Pressure-driven

$$\bar{V} = \frac{gS_f}{v_s}\frac{D^2}{32} \tag{17}$$

Free-surface

$$\bar{V} = \frac{gS_f}{v_s}\frac{h^2}{3} \tag{18}$$

Because of integration, the mean velocity for pressure-driven flow refers to 1D, whereas, for free-surface flow, the integration is made with respect to a unit cross section, and hence, the mean velocity refers to 2D. In the latter case, the mean velocity can be extended to a 1D approach, if the water depth is substituted by the hydraulic radius R.

In turbulent flow, the velocity profile in the viscous sublayer is linear, whereas for the rest of the boundary layer it is logarithmic, i.e.

$$V = \frac{V_*}{\kappa}\ln\left(\frac{z}{z_0}\right) \tag{19}$$

where κ is the Von Karman constant equal to approximately 0.41 (White 1998). Instead of the logarithmic profile of velocity, power-law profiles have been proposed by various researchers (e.g., Cheng 2007; Lee et al. 2013).

The mean velocity in the turbulent layer for pressure-driven and free-surface flow can be expressed as follows:

Pressure-driven

$$\bar{V} = \frac{V_*}{\kappa}\ln\left(\frac{1}{e^{1.5}}\frac{D}{z_0}\right) \tag{20}$$

Free-surface

$$\bar{V} = \frac{V_*}{\kappa}\ln\left(\frac{1}{e}\frac{h}{z_0}\right) \tag{21}$$

The relationship between δ and z_0 can be determined experimentally, using Nikuradse's experimental data for pressure-driven flow in pipes.

By adopting the Darcy-Weisbach equation and taking into account Eqs. (17) and (18), the friction factor f of the Darcy-Weisbach equation is expressed (only for laminar flow) as:

Pressure-driven

$$f = 8\frac{V_*^2}{\bar{V}^2} = \frac{64}{\mathsf{R}_D} \tag{22}$$

Free-surface

$$f = 8\frac{V_*^2}{\bar{V}^2} = \frac{24}{\mathsf{R}_h} \tag{23}$$

where R_D and R_h = the Reynolds numbers for pressure-driven and free-surface flow, respectively.

Likewise, for the turbulent flow, the friction factor f is determined for pressure-driven and free-surface flow taking into account Eqs. (20) and (21) as follows:

Pressure-driven

$$f = 8\frac{V_*^2}{\bar{V}^2} = \frac{8\kappa^2}{\left[\ln\left(\frac{1}{2e^{1.5}}\frac{D}{z_0}\right)\right]^2} \tag{24}$$

Free-surface

$$f = 8\frac{V_*^2}{\bar{V}^2} = \frac{8\kappa^2}{\left[\ln\left(\frac{1}{e}\frac{h}{z_0}\right)\right]^2} \tag{25}$$

General Form of the Equation

As is known there are no clearly defined boundaries between laminar and turbulent flow and between smooth and rough flow. However, there are transition regimes for which the various equations in the related literature are not valid. Several relationships have been proposed to describe the friction factor in these transition regimes (e.g., Ligrani and Moffat 1986; Cheng and Chiew 1998; Yalin and Da Silva 2001). As a consequence, the friction factor can only be determined using conditions of the *if-then-else* type, in combination with the appropriate equations.

The method quoted subsequently is based on the work of Cheng (2008) and offers an explicit equation for the friction factor determination, which can be applied in pressure-driven pipe flows and is valid unconditionally. According to this approach, f can be written using the following mathematical formulation for transitioning between one regime to another:

$$f = f_L^a f_{TS}^{(1-\alpha)b} f_{TR}^{(1-\alpha)(1-b)} \tag{26}$$

where f_L = the laminar flow friction factor; f_{TS} = the turbulent hydraulically smooth flow friction factor; f_{TR} = the turbulent hydraulically rough flow friction factor; and a, b parameters, which are estimated experimentally. The exponent a ranges from 0 (fully turbulent flow) to 1 (fully laminar flow). The exponent

b ranges from 0 (fully turbulent rough flow) to 1 (fully turbulent smooth flow).

The exponents a and b take the following forms:

$$\alpha = \frac{1}{1 + \left(\frac{\mathsf{R}}{\mathsf{R}_{LT}}\right)^m} \tag{27}$$

$$b = \frac{1}{1 + \left(\frac{\mathsf{R}}{\mathsf{R}_{SR}}\right)^n} = \frac{1}{1 + \left(\frac{\mathsf{R}}{\eta\frac{D}{k_s}}\right)^n} \tag{28}$$

where k_s = the roughness height; and m, R_{LT}, η, n should be determined based on Nikuradse's experimental data.

Preliminary Work

In this section, the parameters of a relationship are recalculated, which is similar to Eq. (26), and is designed for free-surface flow. The reason for following this process instead of adopting similar equations from literature is to avoid the propagation of errors due to smoothing approximations.

First, we recalculated the δ-z_0 relationships and derived new ones, which are very similar to those in the literature (e.g., Bates et al. 2005). For hydraulically smooth flow, the following equation is derived as follows:

$$\frac{1}{z_0} \approx 8.94\frac{V_*}{v_s} \approx \frac{100.6}{\delta} \tag{29}$$

whereas for hydraulically rough flow:

$$\frac{1}{z_0} \approx \frac{33.2}{k_s} \tag{30}$$

Regarding the modeling of the friction gradient through the Darcy-Weisbach equation for pressure-driven flow, various relationships have been derived for both smooth and rough flow, which are based on the Nikuradse's experimental data. In the present study, the following relationships have been derived and proposed for smooth and rough flow, which are fitted to that data using the GRG method:

$$\frac{1}{\sqrt{f}} = \left[0.75\ln\left(\frac{\mathsf{R}}{5.37}\right)\right] \tag{31}$$

$$\frac{1}{\sqrt{f}} = \left[0.88\ln\left(6.82\frac{D}{k_s}\right)\right] \tag{32}$$

To derive an explicit relationship, valid under all flow regimes for pressure-driven flow, Cheng's method [Eq. (26)] is followed. The optimization of the required parameters is achieved using the GRG method as well, which produced the following values: $m \approx 8.4$; $R_{LT} \approx 2{,}712$; $\eta \approx 150$; and $n \approx 1.8$. These values are similar to those proposed by Cheng (2008), although Cheng used relationships for smooth and rough flow that are different from Eqs. (31) and (32).

Combining Eqs. (22), (31), (32), (27), and (28), the following general form for the explicit friction factor determination, valid for all conditions for pressure-driven flow, is derived as follows:

$$f = \left(\frac{64}{\mathsf{R}_D}\right)^a \left[0.75\ln\left(\frac{\mathsf{R}_D}{5.37}\right)\right]^{2(\alpha-1)b} \left[0.88\ln\left(6.82\frac{D}{k_s}\right)\right]^{2(a-1)(1-b)} \tag{33}$$

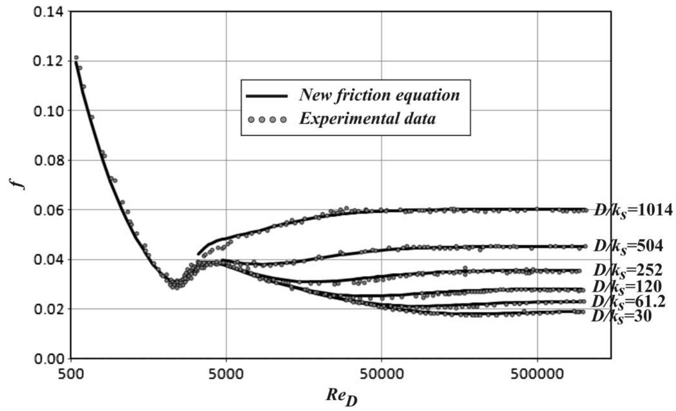

Fig. 6. Comparison between the proposed relationship for pressure-driven flows and experimental data from Nikuradse (1933).

where

$$\alpha = \frac{1}{1 + \left(\frac{\mathsf{R}_D}{2{,}712}\right)^{8.4}} \tag{34}$$

$$b = \frac{1}{1 + \left(\frac{\mathsf{R}_D}{150\frac{D}{k_s}}\right)^{1.8}} \tag{35}$$

The graphical comparison between the results of the preceding equation against Nikuradse's experimental data is presented in Fig. 6.

Derivation of the New Equation for Free-Surface Flow

Cheng (2008) extended his equation, which is similar to Eq. (33), to the case with free-surface flow, assuming $D = 3.2h$ instead of the theoretically expected relationship ($D = 4h$) so as to take into account the lateral embankments of the open channel flow. Because of this, his equation is more appropriate for 1D modeling.

The main disadvantage of this type of approach is that, apart from the laminar part which is physically based via Eq. (23), equations for both the turbulent hydraulically rough and smooth flow require fitting on Nikuradse's data. However, these data are obtained from experiments performed under pressure-driven flow conditions.

In this study, to construct similar relationships for free-surface flow, the relationships between δ and z_0 derived from pressure-driven flow experiments, are used. Therefore, for smooth flow, substituting Eqs. (14) and (29) in Eq. (25), the following equation is derived:

$$\frac{1}{\sqrt{f}} = \frac{1}{\sqrt{8}\kappa}\ln\left(\mathsf{R}_h\sqrt{f}\frac{8.94}{\sqrt{8}e}\right) = 0.86\ln(1.16\mathsf{R}_h\sqrt{f}) \tag{36}$$

which, as implicit, the equation can be solved iteratively by optimization methods such as the Newton-Raphson method. An explicit form can be obtained through the Lambert W function (Corless et al. 1996), which allows for expressing f as follows:

$$\frac{1}{\sqrt{f}} = \frac{\mathsf{R}_h}{0.86e^{W(1.35\mathsf{R}_h)}} \tag{37}$$

where the quantity $W(1.35\mathsf{R}_h)$ is approximated by:

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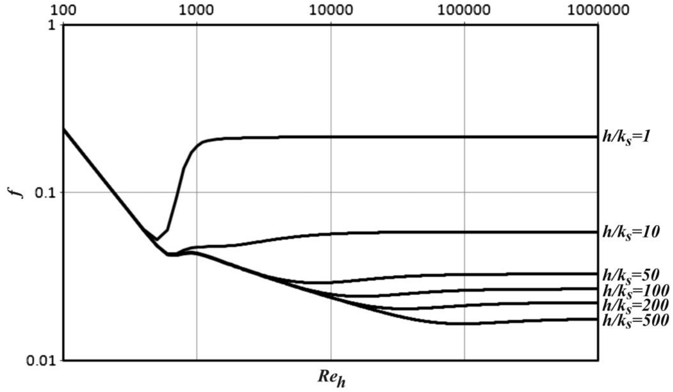

Fig. 7. New friction equation for the free surface flow and for several h/k_s ratios.

$$W(1.35\mathsf{R}_h) = \ln(1.35\mathsf{R}_h) - \ln[\ln(1.35\mathsf{R}_h)] + \frac{\ln[\ln(1.35\mathsf{R}_h)]}{\ln(1.35\mathsf{R}_h)} + \frac{\ln[\ln(1.35\mathsf{R}_h)]^2 - 2\ln[\ln(1.35\mathsf{R}_h)]}{2[\ln(1.35\mathsf{R}_h)]^2} \tag{38}$$

The error of the aforementioned explicit approximation in comparison with the implicit form, solved through the Newton-Raphson method, varies from a maximum value of 1.3% for small Reynolds numbers and a decreasing trend for greater Reynolds numbers R_h.

For the rough flow, substituting Eqs. (14) and (30) in Eq. (25), the following equation is derived:

$$\frac{1}{\sqrt{f}} = \frac{1}{\sqrt{8}\kappa}\ln\left(\frac{33.2}{e}\frac{h}{k_s}\right) = 0.86\ln\left(12.21\frac{h}{k_s}\right) \tag{39}$$

To develop the new friction equation, valid under all regimes and appropriate for free-surface flow, a process that is similar to that for the derivation of Eq. (33) is followed. In this context, the components for smooth and rough turbulent flow have some physical basis, due to the fact that the following assumptions are made: (1) Eqs. (29) and (30) are valid; and (2) exponents a and b are determined using Eqs. (34) and (35).

Therefore, the general explicit equation for the determination of the friction factor, valid under all conditions of free-surface, becomes:

$$f = \left(\frac{24}{\mathsf{R}_h}\right)^{\alpha}\left[\frac{0.86e^{W(1.35\mathsf{R}_h)}}{\mathsf{R}_h}\right]^{2(1-\alpha)b}\left\{\frac{1.34}{\left[\ln\left(12.21\frac{h}{k_s}\right)\right]^2}\right\}^{(1-\alpha)(1-b)} \tag{40}$$

where

$$\alpha = \frac{1}{1+\left(\frac{\mathsf{R}_h}{678}\right)^{8.4}} \tag{41}$$

$$b = \frac{1}{1+\left(\frac{\mathsf{R}_h}{150\frac{h}{k_s}}\right)^{1.8}} \tag{42}$$

The relationship between pipe diameter and hydraulic radius is $D = 4\mathsf{R} = 4h$.

In Fig. 7 the friction factor for the free surface flow is plotted against R_h for a range of values of the relative roughness (h/k_s) using Eq. (40).

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